Synthetic or Real? The Equilibrium Effects of Credit Default Swaps on Bond Markets

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We provide a model of nonredundant credit default swaps (CDSs), building on the observation that CDSs have lower trading costs than bonds. CDS introduction involves a trade-off: it crowds out existing demand for the bond, but improves the bond allocation by allowing long-term investors to become levered basis traders and absorb more of the bond supply. We characterize conditions under which CDS introduction raises bond prices. The model predicts a negative CDS-bond basis, as well as turnover and price impact patterns that are consistent with empirical evidence. We also show that a ban on naked CDSs can raise borrowing costs. (*JEL* G10, G12, G13, G18)

Credit default swap (CDS) markets have grown enormously over the past decade. However, although there is a relatively large literature on the pricing of CDSs, much less work has been done on the economic role of these markets. For example, in most pricing models, CDSs are redundant securities, such that the introduction of a CDS market has no effect on the underlying bond market. This irrelevancy feature makes a meaningful analysis of the economic role of CDS markets difficult.

In this paper, we develop a theory of nonredundant CDS markets, building on a simple, well-documented empirical observation: trading bonds is expensive relative to trading CDSs. Based on this observation, we develop a theory of the interaction of bond and CDS markets. Our model provides an integrated

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framework that can explain many of the stylized facts in bond and CDS markets: the ambiguous effect of CDS introduction on the price of the underlying bond (and therefore financing cost for issuers), the relative pricing of the CDS and the underlying bond (the CDS-bond basis), and trading volume and price impact in the bond and CDS markets. Our model also provides a tractable framework to assess policy interventions in CDS markets, such as the recent EU ban of naked CDS positions.

In our model, investors differ across two dimensions. First, investors differ in their investment horizons: some investors are unlikely to have to sell their position in the future and are therefore similar to buy-and-hold investors, such as insurance companies. Other investors are more likely to receive liquidity shocks and therefore have shorter investment horizons. These latter investors can be interpreted as traders that face redemption risk (e.g., mutual funds), investors who express short-term views, or investors with frequent consumption needs. Second, investors have heterogeneous beliefs about the bond’s default probability: optimistic investors view the default of the bond as unlikely, while pessimists think that a default is relatively more likely. If only the bond is traded, relatively optimistic investors with sufficiently long trading horizons buy the bond, whereas relatively pessimistic investors with sufficiently long trading horizons take short positions in the bond. Investors with short investment horizons stay out of the market, because for them the bond’s trading costs are too high.

The introduction of a CDS affects the underlying bond market through three effects. First, some investors who previously held a long position in the bond switch to selling CDS protection, putting downward pressure on the bond price. Second, all investors who previously shorted the bond switch to buying CDS protection because, in equilibrium, the relatively illiquid bond trades at a discount relative to the CDS. The resulting reduction in short selling puts upward pressure on the bond price. Third, some investors become “negative basis traders” who hold a long position in the bond and purchase CDS protection (i.e., the model endogenously generates the negative basis trade, which has been a popular trading strategy in recent years). If basis traders can take leverage—a natural assumption given that they hold hedged positions—their presence pushes up the bond price. In practice, basis trades are often highly levered and their leverage varies with financial conditions, leading to time-series variation in the strength of this third effect.

Taken together, these three effects imply that the effect of CDS introduction on the underlying bond price is ambiguous. This prediction is consistent with the empirical literature, which has found no unconditional effect of CDS introduction on bond or loan spreads [Hirtle 2009, Ashcroft and Santos 2009]. More important, the model identifies the main economic trade-off associated with CDS introduction. On the one hand, the migration of long and short bond investors to the CDS market (the first and second effects) typically leads to a net crowding-out effect that reduces demand for the bond. On the other
hand, the emergence of basis traders (the third effect) facilitates an allocational improvement in the bond market by allowing long-term investors to absorb more of the bond supply. This basis-trader effect is stronger—and therefore CDS introduction is more likely to increase the bond price—when basis traders can take substantial leverage and when there is a significant trading cost difference between the bond and the CDS.

The endogenous emergence of leveraged basis traders highlights a novel economic role of CDS markets: the introduction of the CDS allows buy-and-hold investors, who are efficient holders of the illiquid bond, to hedge unwanted credit risk in the more liquid CDS market. In the CDS market, the average seller of CDS protection is relatively optimistic about the bond’s default probability, but is not an efficient holder of the bond because of more frequent liquidity shocks. The role of CDS markets is therefore similar to liquidity transformation—by repackaging the bond’s default risk into a more liquid security, they allow the transfer of credit risk from efficient holders of the bond to relatively more optimistic shorter-term investors. Hence, when bonds are illiquid, a liquid CDS can improve the allocation of credit risk and thus presents an alternative to recent proposals that aim at making the corporate bond market more liquid—for example, through standardization (e.g., BlackRock 2013).

Beyond the price effects of CDS introduction, our model generates testable predictions regarding trading volume and price impact in bond and CDS markets that are consistent with recent empirical evidence. First, our model predicts that CDS turnover is higher than bond turnover, consistent with the evidence in Oehmke and Zawadowski (2014), who show that average monthly CDS turnover is above 50%, whereas average monthly turnover in the associated bonds is around 7.5%. Second, our model predicts that CDS introduction decreases turnover in the underlying bond. However, despite this decrease in turnover, CDS introduction can reduce the price impact of supply shocks in the bond market, because levered basis traders act as supply shock absorbers. Therefore, consistent with Das, Kalimipalli, and Nayak (2014), the effect of CDS introduction on bond market liquidity can differ depending on which particular liquidity measure (e.g., turnover or price impact) is used.

From an asset pricing perspective, the prediction that the equilibrium price of the bond is lower than the price of a synthetic bond consisting of a risk-free bond and a short position in the CDS replicates a well-documented empirical phenomenon known as the negative CDS-bond basis (see, e.g., Bai and Collin-Dufresne 2013). Our model generates a number of predictions regarding both the time-series and cross-sectional variation in the CDS-bond basis: the basis is more negative for bonds with higher trading costs, when there is more disagreement about the bond’s default probability and when basis traders are restricted in the amount of leverage they can take.

Finally, our model provides a framework to study regulatory interventions with respect to CDS markets. For example, a ban on naked CDS positions, as recently imposed by the European Union on sovereign bonds through EU
regulation 236/2012, may, in fact, raise yields for affected issuers. If pessimistic investors cannot take naked CDS positions, some of them will short the bond instead. This exerts downward price pressure on bond prices: owing to differences in trading costs, naked CDS positions are not equivalent to short positions in the bond because a different set of investors takes the other side. Similarly, interventions that ban CDS markets altogether, or even both CDSs and short selling of bonds, do not necessarily increase bond prices.

Our paper contributes to a growing literature on derivatives as nonredundant securities. In our framework, a zero-net-supply derivative is nonredundant owing to a difference in the trading costs of the underlying security and the derivative, combined with uninsurable liquidity shocks, which we model using the classic framework of Amihud and Mendelson (1986). Given the well-documented illiquidity of corporate bonds, this source of nonredundancy is likely to be particularly important in the context of the CDS market. The existing literature has focused on different, potentially complementary sources of nonredundancy. Gârleanu and Pedersen (2011) explore the relative pricing of derivatives and underlying assets when derivatives have lower margin requirements and apply this framework to the CDS-bond basis. Shen, Yan, and Zhang (2014) develop a model of financial innovation based on differences in margin requirements. Neither of these two papers focuses on the consequences of derivative introduction on the underlying asset, the main focus of our paper. Banerjee and Graveline (2014) show that derivatives can relax binding short-sale constraints when the underlying security is scarce (“on special”). In their model, the introduction of the derivative always decreases the price of the scarce asset, at least under reasonable investor preferences. Our approach does not rely on explicit short-sale constraints and, therefore, applies also in situations where the underlying asset can be shorted relatively easily. The closest related papers are Fostel and Geanakoplos (2012) and Che and Sethi (2014). Also set in a differences-in-beliefs setup, these papers show that, in the presence of short-sale constraints, naked CDSs can facilitate negative bets that decrease the price of the underlying asset because optimists must set aside collateral to take the other side (see also Geanakoplos 2010 and Simsek 2013). Che and Sethi (2014) also show that when only covered CDS positions are allowed, CDS introduction can increase bond prices because it allows optimists to take leverage (equivalent to collateralized borrowing). Their focus on short-sale constraints and leverage contrasts with our focus on differences in trading costs. These differences in assumptions lead to different predictions: for example, in Che and Sethi (2014), CDS introduction never raises borrowing costs if short selling is possible. In addition, our liquidity-based approach generates predictions on the CDS-bond basis and allows us to study turnover and price impact in the bond and CDS markets, as well as the effect of CDS introduction on different bonds by the

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Hakansson (1969) provides an early discussion of why derivatives should be studied in settings where they are not redundant.
same issuer. Further sources of nonredundancy that have been analyzed include market incompleteness (Detemple and Selden 1991), the informational effects of derivative markets (Grossman 1988; Biais and Hillion 1994; Easley, O’Hara, and Srinivas 1998; and Goldstein, Li, and Yang 2014), the possibility that derivatives generate sunspots (Bowman and Faus 1997), and changes in the relative bargaining power of a firm’s claim holders (Bolton and Oehmke 2011; Arping 2014).

1. Model Setup

We consider a financial market with (up to) two risky assets: (i) a defaultable bond in positive supply and (ii) a CDS that references the bond. The main assumption of our model is that the bond and the CDS, which offer exposure to the same credit risk, differ in trading costs. This difference in trading costs makes the zero-net-supply CDS nonredundant. Specifically, we follow Amihud and Mendelson (1986) and assume that investors incur exogenous trading costs when they trade the bond or the CDS. Our main assumption is that these trading costs are lower for the CDS than the associated bond.

There is strong empirical support for our main assumption. A number of empirical studies have documented high trading costs in the corporate bond market, which contrast with the significantly lower trading costs in CDS markets. This difference in trading costs is driven by a number of factors.


3 For most of the paper, we take the difference in trading costs between the bond and the CDS as given. In Section 3.3.2, we briefly discuss how our model could be extended to account for endogenous trading costs. A number of search-theoretic models have explored endogenous differences in liquidity between assets with identical payoffs (Vayanos and Wang 2008; Vayanos and Weill 2008; and Weill 2008). In this context, Vayanos (2008) studies the interaction between a (liquid) Walrasian and a (less liquid) OTC market, while Samatulana (2014) studies the effect of naked CDS trading in search markets.

4 There are two interpretations of the trading costs in our model. One view is that trading costs are simply transfers to (competitive) dealers. Under this interpretation, trading costs simply reflect the dealers’ costs of efficient liquidity provision and inventory management. Alternatively, trading costs may reflect undersupply of liquidity due to market power of dealers. The positive results in the main part of our paper do not depend on the specific interpretation. However, the source of trading costs matters when drawing normative conclusions (see Section 4).

5 See, e.g., Hessensbiner, Maxwell, and Venkataraman (2006); Edwards, Harris, and Prowse (2007), and Bao, Pan, and Wang (2013). Effective trading costs for bonds include bid-ask spreads and the price impact of trading. Using the most liquid bonds in TRACE, Bao, Pan, and Wang (2013) estimate effective trading costs for corporate bonds of 74–221 basis points. Fletcher, Polito, and Winters (2014) report bid-ask spreads of 4–6 basis points for five-year credit default swaps on IG bonds, which thus implies trading costs of around 20–30 basis points, significantly lower than the effective bond trading costs reported by Bao, Pan, and Wang (2013). Randall (2013) points out that large trades ($10M+) are rare in the bond market and usually have even larger transaction...
First, the CDS market is much more standardized than the bond market, where an issuer’s bonds are usually fragmented into a number of different issues that differ in their coupons, maturities, covenants, embedded options, and so on. Consistent with this argument, Oehmke and Zawadowski (2014) show that CDS markets are more active and more likely to exist for firms whose outstanding bonds are fragmented into many separate bond issues. Second, dealer inventory management is generally cheaper for CDS dealers than for market makers in bond markets: because the CDS is a derivative and can be created at will, there is no need to locate a security and no ex-ante inventory has to be held. Third, a CDS investor who wants to terminate an existing position rarely sells the original CDS in the secondary market; he simply enters an offsetting CDS contract (that can be created), which is usually cheaper.

1.1 Bond
A defaultable bond is traded in positive supply $S > 0$. We denote the bond’s equilibrium price by $p$. The bond matures with Poisson arrival rate $\lambda$. As will become clear below, the assumption of Poisson maturity is convenient because it guarantees stationarity, but none of our results depend on this assumption. At maturity, the bond pays back its face value of $1$ with probability $1 - \pi$. With probability $\pi$, the bond defaults and pays $0$. For simplicity, we assume that the bond does not pay coupons.

We capture illiquidity of the bond market in terms of a bond trading cost $c_B$ that arises when the bond is traded. Specifically, following Amihud and Mendelson (1986), we assume that the bond can be bought at the ask price $p + \frac{c_B}{2}$ and sold (or short sold) at the bid price $p - \frac{c_B}{2}$. The bond price $p$ can therefore be interpreted as the bond’s midquote. The bond trading cost $c_B$ should be interpreted as capturing the bid-ask spread as well as temporary price impact costs that arise, for example, as a result of dealer inventory management. The trading cost $c_B$ is distinct from permanent price impact costs in response to supply shocks, which we derive endogenously later on.

1.2 Credit default swap
In addition to the bond, a CDS that references the bond is available in zero net supply. The CDS is an insurance contract on the bond’s default risk: if the bond defaults, whereas trades exceeding $10M in notional are common in CDS markets. Biswas, Nikolova, and Stahel (2014) provide transaction-based evidence that, in general, CDSs are cheaper to trade than bonds.

This assumption implies that default only occurs at maturity. Alternatively, one could assume that default can occur continuously with Poisson arrival rate. We chose the setup with default only at maturity because it is particularly tractable and yields the same economic insights as a model with continuous default.

Beyond the trading cost $c_B$, which is incurred symmetrically for long and short positions, we do not impose any additional cost on short selling. Although it would be straightforward to add this to the model, treating long and short positions symmetrically highlights that, in contrast to a number of existing papers on CDS or derivative introduction (Banerjee and Graveline 2014; Che and Sethi 2014; and Fostel and Geanakoplos 2012), our results do not require short-sale restrictions.
defaults, the CDS pays out the loss given default of $1 and pays zero otherwise. For simplicity we assume that the CDS matures at the same time as the bond. We denote the CDS’s equilibrium price by $q$. The trading cost in the CDS market is denoted by $c_{\text{CDS}}$, such that an investor can purchase CDS protection at the ask price $q + \frac{c_{\text{CDS}}}{2}$ and sell protection at the bid $q - \frac{c_{\text{CDS}}}{2}$. The CDS price $q$ can therefore be interpreted as the CDS’s midquote.

Our main assumption is that trading costs in the CDS market are lower than in the bond market:

$$c_B \geq c_{\text{CDS}} \geq 0. \quad (1)$$

For most of our analysis, we follow Longstaff, Mithal, and Neis (2005) in assuming, for simplicity, that the CDS market involves no transaction costs, such that $c_{\text{CDS}} = 0$. In Section 3.3.3, we extend our analysis to the case in which the CDS is also subject to trading costs, $c_{\text{CDS}} > 0$.

1.3 Investors

There is a mass of risk-neutral, competitive investors who can trade the bond and the CDS. For simplicity, we set the investors’ rate of time preference to zero. Investors are heterogeneous across two dimensions: (i) expected holding periods and (ii) beliefs about default probabilities. Differences in expected holding periods imply that investors care differentially about trading costs, whereas differences in beliefs about the bond’s default probability generate a motive for trade.

Expected holding periods differ across investors because investors are hit by uninsurable liquidity shocks with Poisson intensity $\mu_i \in [0, \infty)$. Investors with low $\mu_i$ can be interpreted as buy-and-hold investors (for example, insurance companies or pension funds), whereas investors with high $\mu_i$ are investors subject to more frequent liquidity shocks (for example, traders that are exposed to redemption risk, that express shorter-term views, or that face frequent consumption needs). When hit by a liquidity shock, an investor has to liquidate his position and exits the model. To preserve stationarity, we assume that a new investor with the same characteristics enters.

With respect to investor beliefs, we assume that investors agree to disagree about the bond’s default probability in the spirit of Aumann (1976). Specifically, investor $i$ believes that the bond defaults at maturity with probability $\pi_i \in [\bar{\pi} - \frac{1}{2}, \bar{\pi} + \frac{1}{2}]$. These differences in subjective default probabilities among investors lead to differences in valuation of the bond’s cash flows, thereby

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8 In practice, CDS contracts have fixed maturities (also known as tenors), the most common being 1, 5, and 10 years. Our setup, in which both the bond and the CDS randomly mature at the same time, is comparable to a setup in which investors match maturities of finite-maturity bonds and CDSs. Moreover, CDS premia are usually paid over time (quarterly), with a potential upfront payment at inception of the contract. The CDS price $q$ should thus be interpreted as the present value of future CDS premia and the upfront payment.
generating a motive for trade. These differences in valuation of the bond could also be generated by differences in investors’ nontraded endowment risks, which would result in risk-based (rather than beliefs-based) private valuations of the bond. Under this alternative interpretation, \(1 - \pi_i\) represents the risk-based valuation of the cash flows paid by the bond at maturity, based on investor \(i\)’s marginal utilities in the default and nondefault states.

We assume that investors’ beliefs about the bond’s default probability follow a uniform distribution, with a mass one of investors at each liquidity shock intensity \(\mu_i \in [0, \infty)\). This assumption implies a particularly simple conditional density function \(f(\pi | \mu) = \frac{1}{\Delta_1}\), which allows us to calculate equilibrium prices in closed form.

Investors can take positions in the bond and the CDS, but are subject to portfolio restrictions that reflect risk management constraints (given risk neutrality and differences in investor beliefs, absent portfolio restrictions, investors would take infinite positions). Specifically, we assume that each investor can hold up to one unit of credit risk. Accordingly, an investor can go long one bond, short one bond, buy one CDS, or sell one CDS. In addition, investors can enter hedged portfolios. One such option is to take a long position in the bond and insure it by also purchasing a CDS (a so-called negative basis trade). Alternatively, investors can take a hedge position by taking a short position in the bond and selling CDS protection (a so-called positive basis trade). Because hedged positions do not involve credit risk, we allow investors to lever hedged positions to a maximum leverage of \(L \geq 1\). Empirically, this assumption matches the stylized fact that basis traders are usually highly levered. Finally, investors can always hold cash, which yields a zero return.

2. Benchmark: No CDS Market

We first consider the benchmark case in which only the bond trades. In Section 3, we then turn to the joint equilibrium in bond and CDS markets and the effects of CDS introduction.

Investors maximize utility subject to portfolio constraints. When only the bond is trading, this means that investors choose between taking a long or short position in the bond and holding cash. Investor \(i\)’s net payoff from a long position in the bond is given by

\[
V_{\text{long Bond}, i} = -\left( p + \frac{c_B}{2} \right) + \frac{\mu_i}{\mu_i + \lambda} \left( p - \frac{c_B}{2} \right) + \frac{\lambda}{\mu_i + \lambda} (1 - \pi_i) .
\]

9 The differences-in-beliefs setup we use in our model implies that investors do not learn from prices. For models that study the informational consequences of derivatives such as CDSs, see Grosman (1993), Ghian and Hillvio (1999), and Duffie, Li, and Yang (2001).

10 Therefore, \(L = 1\) implies that hedged investors cannot take leverage, whereas \(L > 1\) implies that hedged investors can lever up their positions. Positive leverage for basis traders can be interpreted as the outcome of an (unmodeled) risk management problem: if investors start with the same economic capital, hedging their bets via the CDS allows them to take larger positions against their loss-absorbing capacity. The funding for this is provided by outside investors who are indifferent between holding cash and providing funding to basis traders.
The interpretation of this expression is as follows. The investor pays the ask price $p + \frac{c_B}{2}$ to purchase the bond. With probability $\frac{\mu_i}{\mu_i + \lambda}$, the investor has to sell the bond before maturity. Here, the stationarity property of Poisson maturity implies that a nonmatured bond at some future liquidation date $t$ trades at the same midprice $p$ as the bond today. Hence, the investor receives the bid price $p - \frac{c_B}{2}$ when selling the bond before maturity. If the bond matures before the investor is hit by a liquidity shock, the investor receives an expected payoff of $1 - \pi_i$, where $\pi_i$ is the investor’s subjective belief about the bond’s default probability. This happens with probability $\frac{\lambda}{\mu_i + \lambda}$.

Similarly, investor $i$’s net payoff from a short position in the bond is given by

$$V_{\text{shortBOND},i} = \left( p - \frac{c_B}{2} \right) - \frac{\mu_i}{\mu_i + \lambda} \left( p + \frac{c_B}{2} \right) - \frac{\lambda}{\mu_i + \lambda} \left( 1 - \pi_i \right).$$

An investor who takes a short position in the bond receives the bid price $p - \frac{c_B}{2}$ today. If the investor has to cover his short position before maturity, the investor has to purchase the bond at the ask price $p + \frac{c_B}{2}$ (using stationarity as before), whereas if the bond matures, the investor has to cover his short position at an expected cost of $1 - \pi_i$. The probabilities of these two events are $\frac{\mu_i}{\mu_i + \lambda}$ and $\frac{\lambda}{\mu_i + \lambda}$, respectively.

Figure 1 illustrates the resulting demand for long and short positions. Investors who are optimistic about the bond’s default probability and have sufficiently long trading horizons purchase the bond, forming a triangle of buyers. On the boundary of the “buy” triangle, investors are indifferent between taking a long position in the bond and holding cash, which requires that $V_{\text{longBOND},i} = 0$. Similarly, pessimistic investors with sufficiently long trading horizons short the bond, with the boundary of the resulting “short” triangle defined by $V_{\text{shortBOND},i} = 0$. All other investors hold cash. The gap between the triangle of long bondholders and short sellers arises because the bond trading cost $c_B$ drives a wedge between the payoffs from long and short positions, which makes it optimal even for some investors who do not face liquidity shocks to stay out of the market.

Market clearing requires that the bond price $p$ adjusts such that the overall amount demanded by long bondholders is equal to the amount shorted plus bond supply $S$.

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11 If the bond had finite maturity or if investors updated their beliefs about the bond’s default probability over time, the above valuation equation would be more complicated because the bond would generally trade at a different price at future dates. Nevertheless, the key trade-off would remain: for any price path $p$, investors with more frequent liquidity shocks are affected more strongly by the trading cost $c_B$.

12 We focus on the case in which both long and short positions are taken in the bond-only equilibrium. This requires that the bond trading cost is not so large that it unconditionally rules out short positions, $c_B < \Delta$, and that the bond supply $S$ is not too large. We provide the exact condition on the bond supply in the appendix.
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Figure 1
Bond market equilibrium in the absence of a CDS
The figure illustrates the equilibrium when only the bond is trading. Investors who are sufficiently optimistic about the bond’s default probability and have sufficiently long holding horizons form a “buy bond” triangle. Investors who are pessimistic about the bond’s default probability and have sufficiently long holding horizons form a “short bond” triangle. Market clearing requires that the bond price adjust such that demand from long investors is equal to bond supply plus short positions.

Lemma 1. Benchmark: Bond market equilibrium absent a CDS market.
When only the bond trades, the equilibrium bond price is given by

\[ p_{\text{noCDS}} = \frac{1}{\pi} - \frac{\gamma S}{c_B \lambda \Delta - c_B} \]  

(4)

Lemma[1] shows that, in the absence of the CDS, the bond price is given by the investors’ average belief about the bond’s expected payoff, \( 1 - \pi \), minus a term that captures the bond’s trading costs and supply. Specifically, the term \( -\frac{c_B}{\lambda \Delta - c_B} S \) captures that, as the bond supply \( S \) increases, the marginal bond investor becomes less optimistic and has shorter trading horizons, leading to a decrease in the bond price. Note that the bond trades at a discount relative to the average expected payoff and that the bond price is decreasing in bond trading costs.

3. Introducing a CDS Market
We now introduce the CDS contract into the analysis. As before, we determine the demand for CDS positions by calculating the payoffs from long and short CDS positions, as well as hedged positions in the bond and the CDS. Comparing these payoffs with those from going long or short in the bond, we then solve for equilibrium in the bond and the CDS market.
The net payoff to investor $i$ of purchasing the CDS is given by

$$V_{buyCDS,i} = -\left(q + \frac{c_{CDS}}{2}\right) + \frac{\mu_i}{\mu_i + \lambda} \left(q - \frac{c_{CDS}}{2}\right) + \frac{\lambda}{\mu_i + \lambda} \pi_i. \quad (5)$$

This expression reflects the purchase price $q + \frac{c_{CDS}}{2}$ of the CDS, the payoff $q - \frac{c_{CDS}}{2}$ from early liquidation (using stationarity), and the expected CDS payoff of $\pi_i$ at maturity. As before, $\frac{\mu_i}{\mu_i + \lambda}$ denotes the probability that the investor has to exit his position before CDS payoffs are realized. Analogously, the payoff to investor $i$ of selling CDS protection on the bond is given by

$$V_{sellCDS,i} = \left(q - \frac{c_{CDS}}{2}\right) - \frac{\mu_i}{\mu_i + \lambda} \left(q + \frac{c_{CDS}}{2}\right) - \frac{\lambda}{\mu_i + \lambda} \pi_i. \quad (6)$$

In addition to taking directional positions in the bond or the CDS, investors can enter hedged “basis trade” positions. Taking into account leverage $L \geq 1$, such hedged positions pay off $L(V_{longBOND,i} + V_{buyCDS,i})$ in the case of a negative basis trade and $L(V_{shortBOND,i} + V_{sellCDS,i})$ for a positive basis trade. Finally, investors can still hold cash.

Solving for equilibrium in the bond and CDS markets requires calculating the demand for bond and CDS positions from the above payoffs and then imposing market clearing to determine the equilibrium prices of the bond and the CDS. In our main analysis, we focus on the case in which the CDS market is frictionless ($c_{CDS} = 0$).

### 3.1 The effect of CDS introduction on prices and trading in the bond market

The advantage of assuming that the CDS market is frictionless is that the equilibrium in the CDS market becomes particularly simple: when $c_{CDS} = 0$, Equations (5) and (6) imply that all investors with beliefs $\pi_i < q$ are willing to sell CDS protection ($V_{sellCDS,i} > 0$), while all investors with $\pi_i > q$ are ready to purchase CDS protection ($V_{buyCDS,i} > 0$). Given the infinite support of $\mu_i$, the bond market is then vanishingly small relative to the CDS market, such that the CDS market clears at a price equal to the average investor belief about the bond’s default probability, irrespective of positions in the bond market:

$$q = \bar{\pi}. \quad (7)$$

To determine the equilibrium bond price in the presence of the CDS, it therefore suffices to investigate how the availability of the CDS priced at $q = \bar{\pi}$ affects investors’ incentives to take long or short positions in the bond (i.e., when $c_{CDS} = 0$, we can clear markets sequentially rather than having to solve simultaneously for equilibrium prices in the bond and CDS markets).

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13 Strictly speaking, this is a limit argument: consider an upper bound $\bar{\pi}$ for the frequency of the liquidity shock and then take the limit $\bar{\pi} \to \infty$. As the mass of traders in the CDS market grows, the CDS price $q$ converges to $\bar{\pi}$. 

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This figure illustrates the change in investor strategies that results from CDS introduction when basis traders cannot take leverage \((L = 1)\), holding constant the price of the bond. The dashed lines illustrate the long and short triangles in the absence of the CDS. The introduction of the CDS has three effects: (i) some investors who, absent the CDS, would purchase the bond now choose to sell CDS protection, cutting off the top of the bond-buying triangle; (ii) because the bond trades at a discount relative to the CDS, all former short sellers prefer to purchase CDS protection, which eliminates the shorting triangle; and (iii) investors who formerly bought the bond but whose beliefs about the bond’s default probability are below the average belief \(\bar{\pi}\) become basis traders who purchase the bond and buy CDS protection.

Consider first the case in which basis traders cannot take leverage \((L = 1)\), depicted in Figure 2. The figure shows that the introduction of the CDS results in three effects that change the equilibrium in the bond market. First, when the CDS is available, investors with relatively short trading horizons, who in absence of the CDS used to purchase the bond, now prefer to sell CDS protection. This can be seen in Figure 2. The triangle of long bondholders has been cut off at the top (for ease of comparison, the triangle of long bond positions in the absence of the CDS is depicted by the dashed line). The resulting migration of long bond investors to the CDS market leads to a reduction in demand for the bond, exerting downward pressure on the bond price.

Second, the introduction of the CDS eliminates short selling in the bond. In the figure, the triangle of investors who formerly shorted the bond (depicted by the dotted line on the right) vanishes, because those investors now prefer to purchase CDS protection instead of shorting the bond. The reason why investors prefer to use the CDS market to take negative bets works through the equilibrium price: because of its trading costs, the bond trades at a discount relative to the CDS. Hence, investors who wish to take a bearish bet on the bond prefer to do this in the CDS market rather than through a short position in the bond. By eliminating short sellers, the introduction of the CDS exerts upward pressure on the bond price. Note, however, that the triangle of investors who...
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Figure 3
Bond and CDS market equilibrium (basis traders can take leverage)

The figure illustrates the equilibrium when both the bond and the CDS are trading and basis traders can take leverage ($L > 1$). The ability to take leverage makes the basis trade more attractive, such that the basis-trader triangle expands compared with Figure 2. Because of the increased demand from basis traders, more of the bond can be held by investors with long trading horizons, improving the allocation in the bond market. For ease of comparison, the dashed line illustrates the rectangle of investors who purchase the bond when basis traders cannot take leverage ($L = 1$).

Previously shorted the bond is strictly smaller than the triangle of long bond investors crowded out by the CDS. Under the assumption of a uniform investor distribution, these first two effects therefore lead to a net crowding-out effect that reduces demand for the bond, putting downward pressure on the bond price.

Third, the introduction of the CDS generates a new class of investors: hedged basis traders. Specifically, when $L = 1$ we see that investors who, in the absence of the CDS, would have taken a long position in the bond but whose beliefs about the bond’s default probability is less optimistic than the average belief $\pi$ now find it optimal to purchase both the bond and the CDS. These investors thus become negative basis traders: they hold a hedged position in the bond and the CDS, thereby locking in the equilibrium price difference between the underlying bond and the derivative. Rather than taking bets on credit risk, these investors act as arbitrageurs.

When basis traders cannot take leverage ($L = 1$), as assumed in Figure 2, their presence does not affect the bond price. The reason is that the investors in the basis-trader triangle would have purchased the bond also in the absence of the CDS. When basis traders can take leverage ($L > 1$), on the other hand, the ability to hedge with the CDS allows investors who become basis traders to demand more of the bond. This basis-trader effect therefore exerts upward pressure on the bond price. Figure 3 illustrates that the ability of basis traders...
to take leverage raises the equilibrium bond price in two ways. First, holding constant the number of basis traders (i.e., keeping the size of the basis-trader triangle as in Figure 2), the ability to take leverage increases the demand for the bond from this given set of basis traders. Second, leverage makes the basis trade more profitable and therefore draws more investors into the basis trade: compared with Figure 2, the basis-trader triangle expands. In fact, even some investors to the left of $\pi$ now become basis traders. Even though for these investors the CDS priced at $q = \pi$ has a negative payoff when seen in isolation, they purchase the CDS because it allows them to lever up their position in the bond.  

The endogenous emergence of leveraged basis traders highlights the key economic role of CDS markets in our model: the introduction of the CDS allows buy-and-hold investors, who are efficient holders of the illiquid bond, to hold a larger share of the bond supply and hedge unwanted credit risk in the more liquid CDS market. In the CDS market, the average seller of CDS protection is relatively optimistic about the bond’s default probability, but is not an efficient holder of the bond because of more frequent liquidity shocks. The role of CDS markets is thus similar to liquidity transformation—by repackaging the bond’s default risk into a more liquid security, they allow the transfer of credit risk from efficient holders of the bond to relatively more optimistic shorter-term investors, improving the allocation in the bond market. In particular, separation of credit risk from interest rate risk is possible with an interest rate swap and does not require a CDS. In contrast, the allocational improvement that results from the basis-trader effect is possible only in the presence of a (liquid) CDS contract.

Given the discussion above, we now solve for the equilibrium bond price. Market clearing in the bond market requires that the demand from investors with a long position in the bond and the demand from basis traders add up to the bond supply $S$, given that the CDS market clears at $q = \pi$. Solving for the bond price $p$ that satisfies this market-clearing condition yields the following lemma.

**Lemma 2. Bond price in presence of frictionless CDS market.** When both the bond and a frictionless CDS are traded, the CDS price is given by $q = \pi$ and

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14 When the bond supply or basis-trader leverage is large, it is possible for the basis-trade region to extend all the way to $\pi_i = \pi + \Delta/2$, thereby becoming a trapezoid. Although this would affect some of the analytic expressions calculated below, it would not affect any of the economic predictions of our model. For brevity, we therefore rule out this case. We provide the exact condition in the appendix.

15 When trading costs have a deadweight component, this is an allocational improvement from a welfare perspective. When trading costs are transfers to market makers, then the term “allocational improvement” should be interpreted from the investor’s perspective (i.e., investors incur fewer trading costs).
the equilibrium bond price is equal to

\[ p_{\text{withCDS}} = 1 - \pi - \frac{\Delta \sqrt{1 + 8\Phi \frac{c_B}{2}} - 1}{\Phi} - \frac{c_B}{2}, \]  

(8)

where we define \( \Phi \equiv 1 + 2L(L-1) \).

Similar to Lemma 1, the bond price in the presence of the CDS is equal to the average expected payoff \( 1 - \pi \) and a discount that captures the bond’s trading cost and the fact that, in contrast to the CDS, the bond is in positive supply. In the presence of the CDS, this discount depends on the amount of leverage basis traders can take. In addition, the ability to (synthetically) short the bond via the CDS without incurring a trading cost reduces the bond’s midprice by the cost of setting up a short position in the bond, which is given by the half

spread \( c_B/2 \).

Based on Lemmas 1 and 2, we are now in a position to characterize the effect of CDS introduction on the price of the underlying bond.

**Proposition 1.** The effect of CDS introduction on the bond price. The change in the bond price due to CDS introduction is given by

\[ dp = p_{\text{withCDS}} - p_{\text{noCDS}} = \frac{c_B}{\lambda \Delta} S - \frac{\Delta \sqrt{1 + 8\Phi \frac{c_B}{2}} - 1}{\Phi} - \frac{c_B}{2}. \]  

(9)

(i) The price effect of CDS introduction on the underlying bond is ambiguous.

(ii) The CDS is redundant when \( c_B = 0 \).

(iii) For CDS introduction to raise the bond price, it is necessary that the bond trading cost \( c_B \) and basis-trader leverage \( L \) are sufficiently high.

(iv) CDS introduction is more likely to raise the bond price when disagreement about the default probability \( \Delta \) is small.

Proposition 1 shows that the price effect of CDS introduction is generally ambiguous and depends on the bond trading cost and basis-trader leverage. Setting \( c_B = 0 \), we see that the CDS is redundant when the bond is perfectly liquid. In this case, the CDS has no liquidity advantage over the bond and therefore does not affect the bond price, such that \( p_{\text{withCDS}} - p_{\text{noCDS}} = 0 \).

When \( c_B > 0 \), on the other hand, CDS introduction affects the bond price and the direction of the price effect depends on the balance between the crowding-out effect and the basis-trader effect. As discussed above, under the uniform investor distribution, the migration of (long and short) bond investors to the CDS market leads to a net crowding-out effect. Therefore, CDS introduction invariably reduces the bond price whenever the basis-trader effect is absent, which can happen for two reasons. First, as discussed above, basis traders
do not generate any additional demand for the bond when they cannot take leverage \((L = 1)\). Second, the basis-trader effect becomes negligible when the bond trading cost is close to zero. In this case, the CDS-bond basis approaches zero and the basis-trader triangle is vanishingly small, such that the additional bond demand from basis traders becomes second order, irrespective of their ability to take leverage. In contrast, the migration of bond investors to the CDS market still leads to a first-order reduction in bond demand.\(^{16}\) Although in Proposition 1 we set the CDS trading cost \(c_{\text{CDS}}\) to 0, these results continue to hold when both the bond and the CDS are subject to strictly positive trading costs (see Section 3.3.3).

The above discussion highlights the main economic trade-off that arises when the CDS is introduced. On one hand, CDS introduction can crowd out demand for the bond, which puts downward pressure on the bond price. On the other hand, through the basis-trader effect CDS introduction improves the allocation in the bond market, because it allows investors with long horizons to hold more of the illiquid bond, putting upward pressure on the bond price. As shown above, CDS introduction is more likely to increase the bond price when basis traders can take substantial leverage and when there is a significant liquidity difference between the bond and the CDS.\(^{17}\)

The final result in Proposition 1 states that CDS introduction is more likely to increase the bond price when there is little disagreement about the bond’s default probability. The intuition for this result is that the positive price effect of CDS introduction is mainly driven by investors with moderate beliefs, who become levered basis traders when the CDS is available. An increase in disagreement reduces the mass of investors with moderate beliefs and therefore weakens the increase in bond demand from basis traders.

The main empirical prediction of Proposition 1 is that the price effect of CDS introduction on bond prices is generally ambiguous and depends on bond and firm characteristics. This is consistent with the emerging empirical literature on the effect of CDSs on the cost of financing for firms. Moreover, the specific

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\(^{16}\) The result that CDS introduction always reduces the bond price when the basis-trader effect is not present (i.e., \(L = 1\) or \(c_B \to 0\)) is more general than the uniform investor distribution: it is sufficient to assume that the joint density \(f(\mu, \pi)\) is symmetric around \(\pi\) and satisfies translational invariance with respect to \(\mu\), i.e., \(f(\mu, \pi) = f(\mu', \pi)\). However, relaxing these assumptions, one can find distributions for which the reduction in short selling outweighs the crowding out of long positions. In this case, CDS introduction can raise the bond price even when \(L = 1\) or \(c_B \to 0\). From Figure 2, we see that this requires that the mass of traders in the shorting triangle is larger than the mass of long bond investors crowded out by CDS introduction. Note, however, that the main economic trade-off between (potential) crowding out of bond demand due to migration of investors to the CDS market and the additional demand generated by basis traders remains unchanged.

\(^{17}\) One way to isolate the forces at work is to consider the extreme cases in which only one of the two dimensions of heterogeneity is present. If investors differ only in their trading frequencies and there is no disagreement about default probabilities, then only the basis-trader effect is present: investors with infrequent trading needs hold levered basis positions, purchasing CDS protection from investors with more frequent liquidity shocks. In this case, CDS introduction always raises the bond price. In contrast, when there is only disagreement about the default probability, no basis traders emerge and only the crowding-out effect is present: relatively optimistic investors are indifferent between holding the bond and selling CDS protection, whereas pessimistic investors purchase CDS protection. In this case, CDS introduction lowers the bond price.
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Empirical patterns regarding which issuers benefit from CDS introduction are in line with the predictions of our model: consistent with the prediction that relative trading costs matter, Ashcraft and Santos (2009) and Shim and Zhu (2014) find that CDS introduction tends to reduce funding costs when bond and CDS differ sufficiently in liquidity. Similarly, Nashikkar, Subrahmanyam, and Mahanti (2011) show that, controlling for bond liquidity (measured by their “latent liquidity” measure), bonds of issuers with more liquid CDS contracts (in terms of bid-ask spreads) have lower yields. Consistent with the prediction that CDS introduction is less likely to raise bond prices when there is substantial disagreement, Ashcraft and Santos (2009) find that firms with high earnings forecast dispersion face increased funding costs once a CDS is introduced. Finally, Jiang and Zhu (2015) provide direct evidence for the investor holding patterns predicted by our model: mutual funds with more frequent liquidity needs (proxied by fund flow volatility and portfolio turnover) are more likely to substitute long bond positions with short positions in the CDS.

In addition to the result on the effects of CDS introduction on the bond price, our model generates predictions regarding turnover and price impact in bond and CDS markets. We define turnover in the bond market as bond trading volume divided by the supply of the bond, and CDS turnover as CDS trading volume divided by the notional amount (open interest) of outstanding CDSs.

Proposition 2. Turnover in the bond and CDS market.

(i) Turnover in the CDS market is higher than turnover in the bond market.

(ii) Turnover in the underlying bond decreases when the CDS is introduced.

The predictions in Proposition 2 follow relatively directly from the clientele effect that arises because investors sort themselves into the bond and CDS markets depending on the frequency of their liquidity shocks. Nevertheless, these predictions offer an additional dimension along which the model can be linked to empirical evidence: Consistent with the first prediction in Proposition 2, Oehmke and Zawadowski (2014) document average monthly CDS turnover of more than 50%, whereas average turnover in the underlying bonds is around 7.5% per month. Consistent with the second prediction, Das, Kalimipalli, and Nayak (2014) show that CDS introduction is indeed associated with a decrease in turnover in the underlying bond.

However, even though CDS introduction unambiguously lowers bond turnover, this does not necessarily imply that the bond market becomes less liquid in terms of the permanent price impact of bond supply shocks, \[ \frac{\partial p}{\partial S} \].

1 Ashcraft and Santos (2009) proxy for CDS liquidity using the number of daily quotes, whereas Shim and Zhu (2014) use the standard deviation of CDS quotes.

19 Note that the permanent price impact of bond supply shocks is an endogenous object and therefore distinct from the exogenous bond trading cost \( c_B \), which captures the bid-ask spread and temporary price impact costs.
In fact, CDS introduction can reduce the permanent price impact of bond supply shocks despite lower turnover in the bond market.

**Proposition 3. The effect of CDS introduction on price impact in the bond market.** CDS introduction reduces price impact in response to bond supply shocks, \( \left| \frac{dP}{dS} \right| \), when

(i) basis-trader leverage \( L \) is sufficiently high,
(ii) the bond trading cost \( c_B \) is sufficiently high, and
(iii) disagreement about the bond’s default probability \( \Delta \) is sufficiently low.

Proposition 3 shows that, depending on basis-trader leverage, bond trading costs, and disagreement, CDS introduction can decrease the permanent price impact of bond supply shocks, \( \left| \frac{dP}{dS} \right| \). This effect is driven by the presence of basis traders: levered basis traders cushion the price effect of bond supply shocks, particularly when \( L \) and \( c_B \) are large. This is consistent with the evidence in Massa and Zhang (2012), who show that CDSs dampen the price impact of forced bond sales. Accordingly, the effect of CDS introduction on bond “liquidity” can differ depending on the specific liquidity measure used (e.g., turnover or price impact). Proposition 3 therefore provides a potential explanation for the empirical results of Das, Kalimipalli, and Nayak (2014), who find that, even though CDS introduction reduces bond turnover, there is no clear directional effect of CDS introduction on the Amihud (2002) price impact measure.

### 3.2 The CDS-bond basis

In this section, we investigate the relative pricing of the bond and the CDS when both instruments are available. The relative pricing of bonds and CDSs is captured by the CDS-bond basis, which has attracted considerable attention in the wake of the financial crisis of 2007–09. The CDS-bond basis is defined as the difference between the spread of a synthetic bond (composed of a long position in a risk-free bond of the same maturity and coupon as the underlying risky bond and a short position in the CDS) and the spread of the actual underlying bond. Intuitively speaking, when the CDS-bond basis is negative, the bond spread is larger than the CDS spread, which means that the bond is cheaper than the payoff-equivalent synthetic bond.

Absent frictions, the CDS-bond basis should be approximately zero. The reason is that a portfolio consisting of a long bond position and a CDS that insures the default risk of the bond should yield the risk-free rate. Since the financial crisis, the CDS-bond basis has been consistently negative for many reference entities.\(^{20}\)

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20 See Duffie (1999) for conditions under which this arbitrage relationship holds exactly.

21 See, e.g., Bai and Collin-Dufresne (2013). Note, however, that positive bases do occur in some instances and are usually attributed to frictions that are outside of our model, such as short-selling constraints that arise from...
In our framework, a negative basis between the bonds and the CDS arises endogenously from the difference in trading costs. To calculate the basis, note that in our setting a risk-free bond with the same maturity as the risky bond trades at a price of one (since there is no discounting). We can then calculate the spread of the risky bond above the risk-free rate as the price difference between the risk-free and the risky bond divided by the expected time to maturity $1/\lambda$. This yields a bond spread of $\lambda(1 - p)$. Analogously, given the CDS price $q$ we can calculate the CDS spread as $\lambda q$. The CDS-bond basis, defined at midprices, is then given by

$$\text{basis} = \text{spread}_\text{CDS} - \text{spread}_\text{bond} = -\lambda(1 - p - q).$$  \hspace{1cm} (10)

In addition, we define the “implementable basis,” which includes the cost of establishing the basis-trade position and therefore reflects the basis-trade return available to an arbitrageur, as:

$$\text{implementable basis} = \text{ask} \text{ spread}_\text{CDS} - \text{ask} \text{ spread}_\text{bond} = \text{basis} - \lambda \frac{c_B + c_{\text{CDS}}}{2},$$  \hspace{1cm} (11)

where, as in the preceding analysis, we set $c_{\text{CDS}} = 0$. Based on the bond and CDS market equilibrium derived above, the CDS-bond basis then satisfies the following properties.

**Proposition 4. The CDS-bond basis.** In the presence of a frictionless CDS market ($c_{\text{CDS}} = 0$), the CDS-bond basis is given by

$$\text{basis} = -\lambda \frac{c_B}{2} - \lambda \frac{\Delta}{2} \frac{\sqrt{1 + 8 \Phi \frac{c_B}{\lambda} \frac{S}{\Delta}} - 1}{\Phi} \leq 0,$$  \hspace{1cm} (12)

where $\Phi \equiv 1 + 2L(L - 1)$. The CDS-bond basis (and the implementable CDS-bond basis) is more negative when

(i) bond supply $S$ is large,
(ii) the bond trading cost $c_B$ is high,
(iii) basis traders can take less leverage (small $L$), and
(iv) disagreement about the bond’s default probability $\Delta$ is high.

The source of the negative basis is straightforward. Because bond trading costs are higher than those of the CDS, the bond trades at a discount relative to the CDS. The resulting CDS-bond basis is larger when basis traders find it harder to trade against the basis (small $L$) and when the supply of the bond is large.
Proposition 4 generates a number of time-series and cross-sectional predictions on the CDS-bond basis. First, the basis becomes more negative in response to supply shocks in the bond market, consistent with evidence in Ellul, Jotikasthira, and Lundblad (2011). Second, bonds with high trading costs (relative to the associated CDS) are predicted to have more negative CDS-bond bases, consistent with the evidence in Bai and Collin-Dufresne (2013), who find that higher bond bid-ask spreads are associated with a more negative CDS-bond basis. Third, higher basis-trader leverage compresses the negative basis. Therefore, at times when basis traders can take substantial leverage, the basis should be close to zero (the implementable basis goes to zero as $L \to \infty$).

In contrast, during times of tough funding conditions, the equilibrium basis becomes more negative, consistent with the evidence in Gârleanu and Pedersen (2011), Fontana (2012), and Mitchell and Pulvino (2012). Relatedly, Choi and Shachar (2014) provide evidence that the unwinding of CDS-bond basis arbitrage trades was a main cause of the large negative basis in 2008. Fourth, bonds characterized by substantial disagreement about default probabilities have more negative bases. In practice, high-yield bonds usually have high levels of disagreement and high trading costs. Consistent with our model, they also have more negative CDS-bond bases (Gârleanu and Pedersen 2011, Bai and Collin-Dufresne 2013).

In addition to characterizing the determinants of the CDS-bond basis, our model pins down the size of the basis trade.

**Corollary 1. The size of the basis trade.** The amount of bonds held by basis traders in equilibrium is given by

$$\text{size of the basis trade} = \frac{L (L - \frac{1}{2})}{\Delta \lambda c_B} \text{(implementable basis)}^2.$$  \hspace{1cm} (13)

In equilibrium, basis-trade positions are increasing in the bond supply $S$, the bond trading cost $c_B$, and basis-trader leverage $L$ and are decreasing in disagreement $\Delta$.

The main prediction of Corollary 4 is that the joint dynamics of the CDS-bond basis and the size of the basis trade depend on the type of shock that moves the basis. When the basis becomes more negative because of bond supply shocks or higher bond trading costs, this also makes the basis trade more profitable, drawing more investors into the basis trade. In these cases, a larger negative basis is associated with larger basis-trader positions. In contrast, lower basis-trader leverage and higher disagreement make the basis more negative because they reduce the supply of basis-trader capital, such that a larger negative basis is associated with smaller basis-trader positions. Consistent with these predictions, Oehmke and Zawadowski (2014) document a positive cross-sectional relation between CDS positions (a proxy for CDS demand by basis traders) and bond trading costs. They also show that, as funding conditions
worsen (i.e., basis-trader leverage \( L \) decreases), the correlation between the CDS positions and the implementable negative basis weakens.

### 3.3 Extensions

In this section we discuss how our framework can be extended to allow for multiple bond issues, endogenous trading costs, and positive trading costs in the CDS market.

#### 3.3.1 Two bond issues.

In this subsection, we briefly discuss an extension of our model to a setting where an issuer has multiple bond issues outstanding: a more liquid issue and a less liquid issue. One interpretation of this setting is that the liquid issue is a recently issued “on-the-run” bond, while the less liquid issue is an older “off-the-run” bond. Alternatively, the liquid bond may be a relatively standard bond, whereas the illiquid bond is more custom-tailored for a particular investor clientele. Because the liquid and illiquid bonds are held by different investors, CDS introduction affects the prices of the two bonds differentially. The liquid bond is held by investors with relatively frequent liquidity shocks and is therefore affected disproportionately by the crowding-out effect of CDS introduction. The illiquid bond, on the other hand, benefits disproportionately from the increased demand from basis traders. This implies that illiquid bonds generally benefit more from CDS introduction than liquid bonds. In fact, it is possible that the price of the illiquid bond increases while the price of the liquid bond decreases in response to CDS introduction (see section 1 of the online appendix for further details).

Given that illiquid bonds benefit disproportionately from CDS introduction, firms with CDSs may issue more of these types of bonds. For example, when a CDS is available, firms may issue more customized bonds that cater to specific investors (e.g., pension funds or insurance companies), given that the CDS allows these investors to take levered hedged positions. To the extent that long-term bonds are more illiquid than short-term bonds, CDS introduction may also induce firms to issue more long-term bonds, consistent with the evidence in Saretto and Tookes (2013).

#### 3.3.2 Endogenous trading costs.

An important assumption of our analysis is that the trading costs \( c_B \) and \( c_{CDS} \) are exogenous and, therefore, the bond trading cost \( c_B \) is not affected by CDS introduction. While fully endogenizing trading costs would go beyond the scope of this paper, the results on turnover presented in Proposition 2 allow us to consider endogenous trading costs in reduced form. In search models (e.g., Duffie, Gârleanu, and Pedersen 2005), a reduction in trading activity is usually associated with higher trading costs in the form of larger bid-ask spreads. We can allow for this possibility through a slight

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22 More formally, if CDS introduction reduces the intensity with which bond traders meet trading partners, this would result in lower trading activity and higher bid-ask spreads.
adjustment in Equation (9). In the third term of the expression, $c_B$ would now be replaced by $\tilde{c}_B > c_B$ to reflect higher bond trading costs post CDS introduction. The increase in bond trading costs would therefore put negative pressure on the bond price, partially offsetting the potential bond price increase resulting from CDS introduction.

3.3.3 CDS market with frictions. Up to now, we have focused on the particularly tractable case in which CDSs involve no trading costs ($c_{\text{CDS}} = 0$). This assumption made solving for the equilibrium in the CDS market particularly easy because it ensured that $q = \pi$, which allowed us to solve sequentially for equilibrium prices of the CDS and the bond. When $c_{\text{CDS}} > 0$, this is no longer possible and one has to jointly solve for bond and CDS prices. Because the CDS price now also reflects trading frictions, it is generally the case that $q \neq \pi$. Although closed-form solutions are not available for this case, the main economic results derived above continue to hold (see section 2 of the online appendix for further details).

Figure 4 illustrates the equilibrium when there are also trading costs in the CDS market. In contrast to the frictionless CDS case, investors with sufficiently frequent liquidity shocks now stay out of the market altogether and hold cash. Despite this difference, the introduction of a CDS affects the bond market through the same three effects as before: (i) the CDS crowds out some long bondholders; (ii) the CDS eliminates short sellers; and (iii) the CDS leads to the emergence of hedged basis traders, who exert upward pressure on the bond price when they can take leverage ($L > 1$).

Despite the added complexity of this case, the effect of CDS introduction is similar to the benchmark case discussed in the main part of the paper.

**Proposition 5.** CDS introduction when the CDS market is also subject to trading costs. When the CDS market is also subject to trading costs, $0 < c_{\text{CDS}} \leq c_B$, then

(i) The price effect of CDS introduction on the underlying bond is ambiguous.

(ii) The CDS is redundant when $c_B = c_{\text{CDS}}$.

(iii) For CDS introduction to raise the bond price, it is necessary that both the trading cost advantage of the CDS, $c_B - c_{\text{CDS}}$, and basis-trader leverage $L$ are sufficiently high.

Therefore, the main economic trade-off between the crowding-out effect that reduces demand for the bond and the basis-trader effect that leads to an improvement in the bond allocation is unchanged when we allow for $c_{\text{CDS}} > 0$.

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23 More generally, in such an extension the trading costs in the bond and CDS markets would be given by fixed points, where each trading cost must be consistent with the amount of trading activity in the respective market. Note that such a fixed-point argument may make the trading cost difference between the bond and the CDS self-sustaining, similar in spirit to the results of Vayanos and Wang (2007).
4. Discussion and Policy Implications

4.1 Welfare

Our analysis up to now has focused on positive results—for example, how does CDS introduction affect the price of the underlying bond? In this section, we discuss the extent to which our framework allows us to draw normative conclusions: Does the introduction of a CDS improve welfare?

Welfare effects depend on the interpretation of the differences in valuations that generate trading motives. We first consider the case in which differences in the valuations of the cash flows paid by the bond and the CDS arise because of risk-based private valuations. We then discuss how the welfare conclusions change if trading motives are generated by differences in beliefs. Also note that meaningful welfare discussion should include the issuer’s investment decision. We model this in the simplest possible way by assuming that the issuer sells a fixed amount $S$ of bonds and invests the funds in a technology that pays $R \geq 1$ per unit invested in the no-default state. We focus on total (utilitarian) welfare and assume throughout that trading costs are deadweight, for example, because they reflect market power of dealers.\(^{24}\)

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\(^{24}\)The analysis would be similar if trading costs represent pure transfers. The main difference is that when trading costs are transfers, changes in incurred trading costs are not relevant for overall welfare.
The key to assessing welfare is the observation that, holding fixed the issuer’s investment decision, price changes due to CDS introduction cancel out. They are transfers, either between long and short investors, or between long bondholders and the issuer. Therefore, what matters for welfare is the ability of investors to share risk, the trading costs incurred to do so, and the issuer’s investment decision. Denote an investor’s position in the bond or CDS by $x_{\text{BOND}}$ and $x_{\text{CDS}}$, respectively. In the online appendix, we show that, under risk-based private valuations $\pi_i$ and an objective default probability of $\pi$, total welfare can be written as

$$W_1 = \int_{(\mu, \pi)} \left\{ x_{\text{BOND}}(\mu, \pi) - x_{\text{CDS}}(\mu, \pi) \right\} (1 - \pi) - \left| x_{\text{BOND}}(\mu, \pi) \right| \left( \frac{1}{2} + \mu \lambda \right) c_B - \left| x_{\text{CDS}}(\mu, \pi) \right| \left( \frac{1}{2} + \mu \lambda \right) c_{\text{CDS}}$$

This expression captures the payoffs to investors, trading costs (consisting of the lifetime trading costs of the bond and the CDS and the cost of setting up initial short positions), the net present value (NPV) generated by the issuer’s investment, and the bond payment made by the issuer to investors.

Based on Equation (14), we see that our model highlights four positive welfare effects of CDS introduction. First, the migration of offsetting long-short bond positions to the CDS market is welfare-improving because it lowers the incidence of trading costs that agents pay to share risk (the price at which this risk is shared changes, but this constitutes a transfer). Second, the availability of the CDS improves risk sharing because it allows investors who were previously sidelined by the bond’s high trading cost to share risk via the CDS. Third, the emergence of levered basis traders improves the bond allocation by reducing trading costs incurred by investors who hold the bond and by allowing, via the CDS market, the transfer of credit risk to investors who are more efficient holders of default risk. Fourth, Equation (14) shows that a bond price increase has a positive effect on welfare if it allows the issuer to increase positive NPV investment. Note that, taken together, these findings imply that a bond price increase (our focus in the main part of the paper) is a sufficient condition for an increase in welfare if it allows the issuer to increase positive NPV investment. On the other hand, because of improvements in risk sharing and reductions in trading costs, a decrease in the bond price does not necessarily translate into lower welfare.

When trading motives are caused by differences in beliefs, the main change to the welfare analysis is that offsetting positions now constitute zero-sum bets and do not generate risk-sharing benefits. In this case, the welfare criterion proposed by Brunnermeier, Simsek, and Xiong (2014) implies that all offsetting long-short bets (in the bond or the CDS) matter for welfare only to the extent that they...
lead to the incidence of deadweight transaction costs. In the online appendix, we show that, for a given belief \( \hat{\pi} \) about the issuer’s default probability, total welfare can then be written as:

\[
W_2 = -\int_{(\mu, \pi)} \left\{ \left( x_{\text{BOND}}(\mu, \pi) \left( \frac{1}{2} + \frac{\mu}{\lambda} \right) c_B + x_{\text{CDS}}(\mu, \pi) \left( \frac{1}{2} + \frac{\mu}{\lambda} \right) c_{\text{CDS}} \right) dF(\mu, \pi) \right\} + \left[ (1 - \hat{\pi}) R - 1 \right] pS. \tag{15}
\]

Equation (15) shows that, as before, the migration of long-short bets from the bond to the CDS market is welfare-improving because it lowers the incidence of transaction costs that agents pay to make these bets. However, if there is a trading cost also in the CDS market, this gain from moving existing long-short bets in the bond to the CDS market has to be traded off against the trading costs incurred through additional speculative bets that emerge in the CDS market (in contrast to the risk-based private valuation case, these additional bets no longer generate any risk-sharing benefits). Another important difference is that the welfare consequences of a change in investment in response to a change in the bond price are more difficult to assess. Unless one imposes the restrictive assumption that the issuer’s investment has positive NPV under any reasonable default probability \( \hat{\pi} \), no clear statements can be made.

Finally, note that the above discussion implies that CDS introduction has distributional consequences. For example, when CDS introduction leads to higher bond prices, some bondholders are worse off (from an ex-ante perspective) when the CDS is available.

### 4.2 Bond standardization

The prediction that CDS markets allow buy-and-hold investors to absorb more of the bond supply provides an interesting angle on the recent discussion on standardization in the bond market. For example, in a recent proposal, BlackRock (2013) argues for more standardized corporate bonds, in an attempt to improve secondary-market liquidity (i.e., lowering \( c_B \)). However, as some market participants have pointed out, for issuers standardization may come at the expense of being able to tailor bonds to particular clienteles; a bond that is tailored to a particular investor may fetch a higher at-issue price, despite higher trading costs in the secondary market. Our results suggest that the presence of a CDS allows such issues to be held predominantly by buy-and-hold investors, which reduces the incidence of higher secondary-market trading costs that come with customization. Therefore, the introduction of the CDS can be viewed as

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25 Intuitively, deadweight transaction costs are the equivalent of the destruction of the pillow in the main example of Brunnermeier, Simsek, and Xiong (2014).
a "backdoor" way to achieve some of the benefits of bond standardization, while still allowing issuers to cater their bonds to specific investors.

Our model also highlights that bond standardization, like CDS introduction, has distributional consequences. Proposals to standardize the bond market in order to improve secondary-market liquidity are therefore unlikely to be supported by all market participants. Clearly, dealers may oppose moves toward bond standardization if the resulting reduction in trading costs reduces dealer profits. But bond market participants may also be opposed; in particular, long-term bond investors are negatively affected (in an ex-ante sense) because for them the bond price increase outweighs the reduction in bond trading costs that they incur. This is easiest to see for the extreme case of pure buy-and-hold investors \( \mu_i = 0 \), who pay a higher price for the bond but do not benefit from lower trading costs. Generally speaking, the set of investors that loses from bond standardization is the same set of investors that is negatively affected by CDS introduction that raises the bond price; in both cases, buy-and-hold investors lose out, whereas investors with more frequent trading needs are more likely to gain.

### 4.3 Naked (and other) CDS bans

In this section, we apply our framework to assess a number of policy interventions: (i) banning naked CDS positions (as recently implemented for EU sovereign bonds), (ii) banning naked CDS and short positions in the bond, (iii) banning CDS markets altogether, and (iv) banning both CDSs and short positions in bonds. We mainly take a positive perspective and use our framework to assess these interventions against the policy objective of reducing bond yields, and therefore borrowing costs, for issuers. However, recall from Section 4.1 that, from a welfare perspective, potential benefits from lower yields have to be traded off against restricted risk sharing.\(^\text{26}\)

First, we consider a naked CDS ban. EU regulation 236/2012, in effect since November 1, 2012, allows market participants to purchase CDS protection only if they own the underlying bond or have other significant exposure to the sovereign, thereby restricting so-called naked CDS positions. Short selling of the bond is allowed under this regulation as long as the short seller is able to borrow the bond. The effect of a ban on naked CDS positions depends on what investors who were previously holding naked CDS protection choose to do instead. Our framework highlights three effects, illustrated in Figure 5. First,

\(^{26}\) Clearly, the simple framework proposed here is not rich enough to yield detailed policy prescriptions. Moreover, some of the policies that we discuss in the following subsections may be driven by considerations that are outside of our model. For example, our model does not capture so-called bear raids, which are sometimes cited as a justification for the naked CDS ban in Europe. Nevertheless, even in the context of our simple framework, the effects of CDS market interventions on bond yields are subtle and can potentially go in the “wrong” direction (i.e., contrary to the policymaker’s objective, such interventions can increase borrowing costs for issuers). Also note that, in the context of sovereign bonds, our assumption that the bond has higher trading costs than the CDS implies that the following policy analysis is likely not applicable for the most liquid sovereign bonds, such as U.S. Treasury bonds or German Bunds.
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Figure 5
Banning naked CDS when CDS > 0 and L > 1

The figure illustrates the change in investor strategies when naked CDS positions are banned, holding constant the price of the bond. The dashed line shows the position of CDS buyers and basis traders before the ban. Compared with Figure 4, which depicts the same setup except that naked CDS positions are allowed, there are two major changes. Some investors who used to purchase naked CDS protection now choose to short the bond, exerting downward pressure on the bond price. Some investors who used to purchase naked CDS protection now become basis traders, which exerts upward pressure on the bond price.

Some investors switch from a naked CDS position to a short position in the bond. Hence, as a result of a ban on naked CDSs, short sellers reappear, putting downward pressure on the bond price. Second, some investors who formerly held a naked CDS position become basis traders and hold the bond and the CDS, up to the maximum leverage L. This second effect increases demand for the bond, resulting in upward pressure on the bond price. Third, some investors who previously held naked CDS protection switch to simply holding cash.

The effect of banning naked CDS positions on the cost of borrowing therefore depends on the relative size of these effects and, in the absence of restrictions on the investor distribution, can go in either direction. In particular, it is possible that bond and CDS spreads move in opposite directions in response to a naked CDS ban such that borrowing costs for issuers may rise, even when CDS spreads decline. Consistent with this prediction, a recent report by the European Securities and Markets Authority on the naked CDS ban in Europe [ESMA 2013] documents a modest 26-basis-point reduction in CDS spreads, but finds no evidence that EU sovereign bond yields dropped as a result of the naked CDS ban.

Second, a combined ban on naked CDSs and short positions always raises the bond price in our framework. In particular, as seen above, by itself a naked CDS ban can be ineffective in lowering bond yields because of the reemergence of short sellers in the bond market. In order to guarantee a reduction in borrowing...
costs, a naked CDS ban has to be supplemented with restrictions on short selling in the bond market.

Third, an outright ban of the CDS market amounts to a simple comparison of the equilibrium with a CDS market to the equilibrium without a CDS market. From Proposition 1, we know that the effect of CDS introduction on the bond yield is ambiguous. Therefore, banning CDS markets altogether may either increase or decrease borrowing costs for issuers, depending on parameters. Accordingly, a ban on CDSs is more likely to lead to a reduction in funding costs if trading costs in the bond and the CDS market are similar, and when basis traders are restricted in the amount of leverage they can take.

Fourth, we consider the effect of banning both the CDS market and short positions in the bond. This intervention amounts to a comparison of the bond and CDS market equilibrium described in Proposition 1 to a setting where only long positions in the bond are allowed and no CDS is available. Perhaps surprisingly, this intervention does not necessarily lower bond yields for issuers. Although restricting short positions prevents the reemergence of short sellers in response to a ban on CDS positions, a trade-off now emerges from the countervailing effects of (i) increased demand for the bond from investors who formerly sold the CDS but now purchase the bond and (ii) the reduction in demand for the bond that results from the elimination of basis traders. Because basis traders are price neutral when they cannot take leverage ($L = 1$), in this case a joint ban on CDSs and short selling leads to an unambiguous decrease in the bond yield. When basis traders can take leverage, on the other hand, bond yields may increase or decrease, depending on the relative size of the two effects.

5. Conclusion

This paper provides a liquidity-based model of CDS markets, bond markets, and their interaction. In our framework, CDSs are nonredundant because they have lower trading costs than the underlying bonds. Our model shows that CDS introduction involves a trade-off; it reduces demand for the bond (the crowding-out effect) but leads to an improvement in the bond allocation because it allows long-term investors to become levered basis traders and absorb more of the illiquid bond (the basis-trader effect).

CDS introduction is more likely to raise the price of the underlying bond when there is a significant trading-cost difference between the bond and the CDS and when hedged basis traders can take substantial leverage. For firms with multiple bond issues, the more illiquid bonds (such as off-the-run bonds or custom-tailored issues) are more likely to benefit from CDS introduction.

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27 This long-only case can be solved analogously to the no-CDS case in Lemma 1. The main difference is that the shorting triangle in Figure 1 would disappear. Market clearing then requires that demand from the buying triangle be equal to bond supply.
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Beyond characterizing the impact of CDS introduction on the pricing of the underlying bond, the model also generates empirical predictions regarding trading volume in bond and CDS markets, as well as the cross-sectional and time-series properties of the CDS-bond basis, thereby providing an integrated framework that matches many of the stylized facts in bond and CDS markets. Finally, our framework can be used to assess a number of policy measures related to CDS markets, such as the recent EU ban on naked CDS positions.

To conclude, it is worth pointing out that the main insights from our model may apply to other derivatives that have trading costs that are lower than those of the underlying asset. Beyond CDSs, examples may include index futures, bond futures, and exchange-traded funds (ETFs).

Appendix: Proofs

Parametric assumptions for closed-form solutions. We make three main parametric assumptions in order to simplify the analysis. Our qualitative results do not depend on these assumptions, but relaxing these assumptions would lead to slightly different expressions. Assumptions 1 and 2 ensure that both long and short bond positions are present before CDS introduction (and, hence, the region of long bond investors is a triangle):

**Assumption 1.** \( \Delta > c_B \).

**Assumption 2.** \( S < \frac{1}{2} (\lambda - \Delta c_B)^2 \).

Assumption 3 ensures that the region of basis traders forms a triangle, which requires that basis-trader leverage is not too high:

**Assumption 3.** \( S < \frac{1}{2} \Delta \frac{L^2 + 1}{4} \).

Proof of Lemma 1. It follows from Assumptions 1 and 2 that both long and short bond positions emerge. Moreover, the regions of long and short investors are triangles, as depicted in Figure 1. Evaluating the zero-valuation line of a long bond position, \( V_{\text{longBOND},i} = 0 \) at \( \mu_i = 0 \) and \( \pi_i = \pi - \frac{\Delta}{2} \) yields a right-angled “buy” triangle with base \( 1 - p - \frac{c_B}{2} - (\pi - \frac{\Delta}{2}) \) and height \( \frac{1}{\lambda} (1 - p - \frac{c_B}{2} - (\pi - \frac{\Delta}{2})) \). Similarly, evaluating the zero-valuation line of a short bond position, \( V_{\text{shortBOND},i} = 0 \) at \( \mu_i = 0 \) and \( \pi_i = \pi + \frac{\Delta}{2} \) yields a right-angled “short” triangle with base \( \pi + \frac{\Delta}{2} - (1 - p + \frac{c_B}{2}) \) and height \( \frac{1}{\lambda} \left[ \pi + \frac{\Delta}{2} - (1 - p + \frac{c_B}{2}) \right] \).

Given the uniform conditional density of investors, \( f(\pi | \mu) = \frac{1}{\lambda} \), market clearing then requires that

\[
\frac{1}{\Delta} \left[ 1 - \frac{1}{2} \frac{\Delta}{c_B} \left( 1 - p_{\text{noCDS}} - \frac{c_B}{2} - \left( \pi - \frac{\Delta}{2} \right) \right)^2 - \frac{1}{2} \frac{\Delta}{c_B \Delta - c_B} \left( 1 - p_{\text{noCDS}} + \frac{c_B}{2} \right)^2 \right] = S, \quad (A1)
\]

which yields

\[
p_{\text{noCDS}} = 1 - \frac{c_B}{\lambda} \frac{\Delta}{\Delta - c_B} S. \quad (A2)
\]

Proof of Lemma 2. We first show that the equilibrium CDS price is given by \( q = \pi \), irrespective of positions taken in the bond market. This result allows us to solve sequentially for price of the CDS and the bond. Formally, the pricing of the CDS follows from a limit argument. Suppose that the support of liquidity shock intensities is given by \([0, \pi]\), where \( \pi \) denotes the maximum
liquidity shock intensity. Denote the associated CDS price by \( q(\pi) \). Because of the presence of long bondholders, market clearing in the CDS market requires that, for any finite \( \pi, q(\pi) > \pi \). Intuitively, because long positions take away from potential CDS sellers, the CDS price has to be slightly more attractive than the average default probability for markets to clear. However, in the limit \( \pi \to \infty \), bond positions become negligibly small relative to positions in the CDS market, which implies that the price of the CDS converges to the average default probability, \( \lim_{\pi \to \infty} q = \pi \).

When a CDS priced at \( q = \pi \) is available, solving \( \pi_{\text{longCDS}} > \pi_{\text{longBOND,}} \) for \( \mu_i \) yields that any investor with liquidity shock intensity \( \mu_i > \frac{c_B}{2} (1 - p - \frac{c_B}{2} - q) \) strictly prefers selling a CDS to taking a long position in the bond. Moreover, because of its trading costs, the bond must trade at a price below \( 1 - \pi \). Therefore, given availability of the CDS priced at \( q = \pi \), no investors will short the bond and, by the same observation, the positive basis trade is not profitable.

Comparing the payoff from a negative basis trade, \( L(\pi_{\text{longBOND,}} + \pi_{\text{longCDS,}}) \), with \( \pi_{\text{longBOND,}} \) and \( \pi_{\text{longCDS,}} \), yields a right-angled basis-trader triangle with base \((2L - 1)(1 - p - \frac{c_B}{2} - q)\) and height \( \lambda - (1 - p - \frac{c_B}{2} - q) \). Assumption (A3) guarantees that this basis-trader region is indeed a triangle.

Market clearing in the bond market requires that the demand from long bond investors (the “buy bond” trapezoid) and basis traders (the “basis-trader triangle”) equals the supply of the bond:

\[
\frac{1}{\Delta} \left[ q - \left( \frac{\pi - \Delta}{2} \right) + q - (L-1) \left( 1 - p - \frac{c_B}{2} - q \right) - \left( \frac{\pi - \Delta}{2} \right) \right] \times \left[ \frac{\lambda}{2} \left( 1 - p - \frac{c_B}{2} - q \right) \right] \times \frac{1}{\Delta} \left( 2L - 1 \right) \left( 1 - p - \frac{c_B}{2} - q \right) = S, \tag{A3}
\]

which, substituting in for the equilibrium CDS price \( q = \pi \) and defining \( \Phi = 1 + 2L(L-1) \), yields

\[
p_{\text{withCDS}} = 1 - \pi - \frac{\Delta}{2} \sqrt{\frac{1 + 8\Phi \frac{\Delta}{2} \frac{\lambda}{2} - 1}{\Phi}} - \frac{c_B}{2}. \tag{A4}
\]

**Proof of Proposition** (ii) The bond price change in response to CDS introduction can be calculated directly from Lemmas (i) and (ii).

\[
dp = p_{\text{withCDS}} - p_{\text{noCDS}} = \frac{c_B}{2} \frac{\Delta}{\Delta - c_B} S - \frac{\Delta}{2} \sqrt{\frac{1 + 8\Phi \frac{\Delta}{2} \frac{\lambda}{2} - 1}{\Phi}} - \frac{c_B}{2}. \tag{A5}
\]

Part (i) follows from the observation that (A5) cannot be signed unless we impose further restrictions on parameters (in the proof of part (iii), we provide specific examples of both increases and decreases in the bond price in response to CDS introduction). Part (ii) follows directly from setting \( c_B = 0 \) in (A5), which yields \( dp = 0 \). To show part (iii), we first show that, for a given level of basis-trader leverage \( L \), the bond price decreases in response to CDS introduction when \( c_B \) is sufficiently small. This can be seen by differentiating Equation (A5) with respect to the bond trading cost and evaluating the resulting expression at \( c_B = 0 \), which yields \( -\frac{1}{2} < 0 \). Given that the bond price is not affected by CDS introduction when \( c_B = 0 \), this implies that for bond trading costs close to zero CDS introduction reduces the bond price. To show that the bond price can only increase if basis-trader leverage is sufficiently high, we observe from (A5) that \( p_{\text{withCDS}} - p_{\text{noCDS}} < 0 \) when \( L = 1 \) and that

\[
\frac{d(p_{\text{withCDS}} - p_{\text{noCDS}})}{dL} = \frac{\Delta(2L - 1) \left( -\sqrt{\Delta \lambda (8(2L^2 - 2L + 1)S + \Delta \lambda)} + 4(2L^2 - 2L + 1)S + \Delta \lambda \right)}{(2L^2 - 2L + 1)^2 \sqrt{\Delta \lambda (8(2L^2 - 2L + 1)S + \Delta \lambda)}} > 0. \tag{A6}
\]

From Lemma (i) we see that \( p_{\text{withCDS}} |_{L \to \infty} = 1 - \pi - \frac{\Phi}{2} \). Therefore, if \( p_{\text{noCDS}} < 1 - \pi - \frac{\Phi}{2} \), CDS introduction raises the bond price when \( L \) is sufficiently high. This is the case whenever \( S > \frac{1}{2} \frac{\Delta - c_B}{\Delta} \).
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which is not ruled out by either Assumption [3]. To show (iv), we note that

\[
\frac{d(p_{\text{with CDS}} - p_{\text{no CDS}})}{d\Delta} = \frac{\sqrt{\Delta} \left( 2 \left( 2L^2 - 2L + 1 \right) \right) S_{CB} + \Delta \lambda}{2 \sqrt{\lambda} \left( 2L^2 - 2L + 1 \right)} - \frac{S_{CB}^2}{\Delta \lambda^2} < 0. \tag{A7}
\]

The above condition holds if and only if

\[
\sqrt{\Delta \lambda} \left( 8 \left( 2L^2 - 2L + 1 \right) S_{CB} + \Delta \lambda \right) \left[ \lambda \left( \Delta - c_B \right)^2 - 2 \left( 2L^2 - 2L + 1 \right) S_{CB} \right] < \lambda \left( \Delta - c_B \right)^2 \left[ 4 \left( 2L^2 - 2L + 1 \right) S_{CB} + \Delta \lambda \right]. \tag{A8}
\]

which can be shown to hold by bounding the left-hand side from above by dropping the negative term \(-2 \left( 2L^2 - 2L + 1 \right) S_{CB}^2\). □

Proof of Proposition 2.

To prove (i), we note that trading frequency of all investors selling the CDS is higher than the trading frequency of any investor buying the bond through either a long-only trade or a basis trade (see Figure 2). This implies that turnover generated by CDS sellers (their average trading frequency) is higher than turnover in the bond market (the average trading frequency of investors who hold the bond). Note that this argument even ignores additional CDS turnover generated by CDS buyers (i.e., the turnover generated by CDS sellers strictly underestimates overall CDS turnover).

To prove (ii), we observe that CDS introduction changes the bond-holding regions in two ways, both of which lead to lower bond turnover (see Figure 2). First, the elimination of the shorting triangle unambiguously decreases bond trading. Because the amount of bonds outstanding \(S\) is unchanged, this decreases bond turnover. Second, of the remaining bond buyers (including basis traders) even those with the highest trading frequency have a lower trading frequency than the bond buyers that have been eliminated through introduction of the CDS. Because the overall required number of bond buyers decreases (the CDS eliminates short selling), the mass of low turnover investors added to the bond buyers (if any) is smaller than the mass of former bond buyers who are crowded out into the CDS market. Because these new bond buyers all have a lower trading frequency than the bond investors crowded out by the CDS market, the amount of equilibrium trading diminishes. Given that the bond supply \(S\) is unchanged, turnover in the bond market decreases. □

Proof of Proposition 3.

To compare price impact with and without the CDS, we can use the expressions in Lemmas 1 and 2 to calculate

\[
\left| \frac{dp_{\text{no CDS}}}{dS} \right| = \frac{c_B \Delta}{\lambda \Delta - c_B}, \tag{A9}
\]

\[
\left| \frac{dp_{\text{with CDS}}}{dS} \right| = \frac{c_B \Delta}{\lambda \Delta - c_B} \frac{2}{\sqrt{1 + 8 \Phi \frac{c_B}{\lambda} \Delta}}, \tag{A10}
\]

This implies that price impact is lower in the presence of the CDS if

\[
\frac{\Delta}{\Delta - c_B} > \frac{2}{\sqrt{1 + 8 \Phi \frac{c_B}{\lambda} \Delta}}. \tag{A11}
\]

where, as before, \(\Phi = 1 + 2L(L - 1)\). The results in the proposition then follow directly from (A11). First, note that the right-hand side goes to zero as \(L \to \infty\), whereas the left-hand side is positive and independent of \(L\). Hence, (A11) is satisfied if basis-trader leverage \(L\) is sufficiently high, proving (i). Second, as \(c_B\) increases toward its upper bound \(\Delta\), the left-hand side diverges to
The comparative statics in the proposition follow directly from differentiating the basis with respect to the size of the basis trade multiplied by the leverage parameter $L$, which can be calculated as the mass of traders in the basis-trader triangle multiplied by the leverage parameter $L$. This can be calculated as the mass of traders in the basis-trader triangle multiplied by the leverage parameter $L$, which can be rearranged to yield the expression in the proposition. The comparative statics for $\Delta$ follow directly from differentiating the basis with respect to the relevant parameters.

**Proof of Corollary 1** The size of the basis trade is defined as the amount of bonds (and, equivalently, CDSs) that basis traders own in equilibrium. This can be calculated as the mass of traders in the basis-trader triangle multiplied by the leverage parameter $L$. This yields

\[
\text{size of the basis trade} = \frac{L(2L-1)\sqrt{\Delta (8(2L^2-2L+1)S B + \Delta \lambda) - \Delta \sqrt{\lambda}}}{8\Delta (2L^2-2L+1)^2 c_B},
\]

where, as before, $\Phi \equiv 1+2L(L-1)$. The implementable basis (defined at ask prices) is given by

\[
\text{implementable basis} = -\lambda \frac{\Delta}{2} \sqrt{1+8\Phi \frac{2L+\pi}{\pi} - 1} \leq 0.
\]

The comparative statics in the proposition follow directly from differentiating the basis with respect to $S, c_B, L$, and $\Delta$ and are omitted for brevity. ■

**Proof of Proposition 5** The size of the basis trade is defined as the amount of bonds (and, equivalently, CDSs) that basis traders own in equilibrium. This can be calculated as the mass of traders in the basis-trader triangle multiplied by the leverage parameter $L$. This yields

\[
\text{size of the basis trade} = \frac{L(2L-1)\sqrt{\Delta (8(2L^2-2L+1)S B + \Delta \lambda) - \Delta \sqrt{\lambda}}}{8\Delta (2L^2-2L+1)^2 c_B},
\]

which can be rearranged to yield the expression in the proposition. The comparative statics for $S, c_B, L$, and $\Delta$ follow directly from differentiating this expression with respect to the relevant parameters. The comparative statics for $L$ are slightly more complicated: The size of the basis trade is increasing in $L$ if and only if

\[
\left(4L^4 - 12L^3 + 12L^2 - 6L + 1\right) \frac{8SB}{\Delta^2} < \left(8L^3 - 6L^2 - 2L + 1\right) \left(1+\frac{2L^2-2L+1}{\Delta} \left(\frac{8SB}{\Delta^2} - 1\right) \right).
\]

For $L \geq 1$ this can be shown to hold if $\frac{SB}{\Delta^2} < 1$, which is true by Assumption 3. ■

**Proof of Proposition 5** When $c_{\text{CDS}} > 0$, closed-form solutions for the equilibrium prices are only available in special cases. Part (i) follows because, in the absence of further restrictions on parameters, the price effect of CDS introduction can go either way (in the proof of part (ii), we provide specific examples of both increases and decreases in the bond price in response to CDS introduction). To show part (ii), we first note that, when $c_{\text{CDS}}$ is sufficiently close to $c_B$, there are no basis traders (the negative basis is smaller than the trading cost to set up the basis trade). Given that there are no basis traders in this case, we can then solve for the equilibrium prices in closed form:

\[
p_{\text{withCDS}} = 1 - \bar{p} - \frac{c_B}{2} + \frac{c_{\text{CDS}}(\Delta - c_{\text{CDS}})}{4(c_B - c_{\text{CDS}})} - \frac{2c_B - c_{\text{CDS}}}{4(c_B - c_{\text{CDS}})} \left(\frac{\Delta - c_{\text{CDS}}}{\lambda} + 8\Delta \frac{S}{\lambda} (c_B - c_{\text{CDS}})\right)
\]

\[
q = \bar{q} - \frac{c_{\text{CDS}}(\Delta - c_{\text{CDS}})}{4(c_B - c_{\text{CDS}})} + \frac{c_{\text{CDS}}}{4(c_B - c_{\text{CDS}})} \left(\frac{\Delta - c_{\text{CDS}}}{\lambda} + 8\Delta \frac{S}{\lambda} (c_B - c_{\text{CDS}})\right).
\]

Note that taking the limit of $\frac{S_B}{\Delta^2}$ as $c_{\text{CDS}} \to c_B$, we recover Equation 4. This shows that the CDS is redundant when CDS and bond trading costs are equal, establishing (ii). Holding
fixed \( L \) and differentiating (A16) with respect to \( c_{\text{CDS}} \) and evaluating the derivative at \( c_{\text{CDS}} = c_B \) yields

\[
dp_{\text{with CDS}} \bigg|_{c_{\text{CDS}} = c_B} > 0,
\]

which establishes that a small reduction of CDS trading costs starting from \( c_{\text{CDS}} = c_B \) always reduces the bond price, establishing the first part of (iii). Finally, it follows from the observation that basis traders are price neutral when they cannot take leverage, that basis-trader leverage \( L \) must be sufficiently high for CDS introduction to increase the bond price. Closed-form solutions are available when \( L \to \infty \). In this case, the basis-trader region shrinks to a point, demanding a finite amount of the bond and an equal amount of the CDS. The implementable CDS-bond basis \( p_{\text{with CDS}} = 1 - q \), but, because of trading costs in the CDS, \( q \neq \pi \). From the equilibrium prices (omitted for brevity), one then finds that CDS introduction raises the bond price whenever

\[
S > \frac{2}{\lambda} \frac{\Delta - \varphi}{\Delta} \left( \frac{\Delta - \varphi}{\Delta} + \frac{\Delta_{\text{CDS}}^2}{\Delta_{\text{CDS}}} \right),
\]

which converges to the condition given in the proof of Proposition

\[
S > \frac{2}{\lambda} \frac{\Delta - \varphi}{\Delta}, \quad \text{when } c_{\text{CDS}} \to 0.
\]

References


Synthetic or Real? The Equilibrium Effects of Credit Default Swaps on Bond Markets


Thompson, J. R. 2009. Credit risk transfer: To sell or to insure. Working Paper, University of Waterloo.


