Entangled Financial Systems

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I model an entangled financial system in which banks hedge their portfolio risks using over-the-counter (OTC) contracts. However, banks choose not to hedge counterparty risk, and thus the idiosyncratic failure of a bank can lead to a systemic run of lenders. An inefficiency arises because banks engage in a version of risk shifting through the network externalities created by OTC contracts. Banks do not take into account that the costly hedging of low-probability counterparty risk also benefits other banks. In the model, it is welfare improving to tax OTC contracts to finance a bailout fund. (*JEL* G22, G28)

Modern financial institutions are entangled in a network of bilateral hedging contracts, such as over-the-counter (OTC) derivatives. The size of the OTC markets is vast: In 2011 the notional amount outstanding exceeded \$600 trillion USD.[1] The fear that these instruments might cause a global financial crisis was a major argument brought up to support government intervention in the financial crisis of 2008.[2] This raises the question addressed in this paper: How can OTC derivates create systemic risk? I show that OTC contracts have a double role: First, they allow idiosyncratic risk to become systemic by creating a contagion channel. Second, they create externalities that undermine the incentives of financial institutions to avert contagion.

In the paper, I develop a model to analyze such an "entangled financial system." There are two types of risks: asset risk and counterparty risk. Each financial institution (bank from hereon) invests in a risky real asset but can

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[1] "Statistical release: OTC derivatives statistics at end-December 2011," Bank of International Settlements, May 9, 2012.

[2] "Wall Street's crisis," *The Economist,* March 22, 2008.

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hedge most of its risks with its neighbors using OTC contracts. These, along with short-term debt, allow banks to economize on costly equity. Thus, banks are entangled in a highly leveraged circular network of OTC contracts in which they hedge their asset risks with their neighbors. Counterparty risk is modeled as a nonzero probability exogenous failure of one of the banks in the system. The failure of this initial bank can drag down neighboring banks but only if they are unprepared, leading to a contagious financial system. I allow banks to hedge counterparty risk by holding more equity, buying default insurance on their counterparties, or collateralizing OTC contracts. I show that under certain conditions, banks do not manage counterparty risk in equilibrium. This result is in line with the empirical observation that large creditworthy institutions do not in general manage counterparty risk among each other.[3]

Buying default insurance on counterparties yields a socially optimal outcome, but it is not sustainable in equilibrium. Counterparty default insurance can be implemented by a contract that resembles a credit default swap (CDS): Banks collectively pay for setting aside equity ex ante that can be deployed in a state contingent manner ex post to the counterparties of the failing bank. Equity injected in the counterparties can prevent the contagion. In equilibrium, banks choose to expose themselves to inefficiently large counterparty risk, even though they know the system might collapse with nonzero probability. The inefficiency comes from risk shifting through network externalities created by the OTC contracts. Banks do not take into account that their own failure can also drag down others: their counterparties, the counterparties of their counterparties, etc. These externalities lead to a market failure: The public good of financial stability is not provided in equilibrium. This inefficient contagious equilibrium exists for an intermediate range of probabilities of counterparty failure. In one extreme, if counterparty defaults are very rare, it is socially optimal not to insure against them. In the other extreme, if counterparty failure is very likely, then it is even individually optimal for banks to insure against counterparty default. In an extension of the model, I show that banks bilaterally demanding counterparty insurance from each other cannot resolve the inefficiency: Banks would not necessarily enforce such a clause. The result that banks fail to buy counterparty default insurance implies that there is not many CDS's in the market written on banks.[4]

This paper helps understand why banks hedge asset risks using OTC contracts thereby creating a channel for contagion but fail to hedge counterparty risk making the system contagious. On the one hand, banks have strong incentives to hedge asset risk using OTC contracts, because not hedging substantially

[3] Singh (2010) estimates that the undercollateralization among large financial institutions is up to $2 trillion.

[4] This is in line with empirical evidence of Oehmke and Zawadowski (2012) that there is relatively little CDS written on financial institutions.

raises their probability of default and default is costly. On the other hand, buying counterparty insurance and thus paying for setting aside costly equity for a low-probability event of being affected by counterparty default is only worthwhile if the probability of counterparty collapse is relatively high.[5] A further reason (outside my simple model) that makes insuring counterparty risk especially costly, is that counterparty defaults tend to happen when a bad aggregate shock hits the economy and equity is scarce. The externalities created by the OTC contracts imply very low incentives to insure counterparty risk, leading to the collapse of the counterparty insurance market. I also show that holding excess equity and collateralization is a wasteful use of scarce equity and is in general not chosen by banks.

The model also highlights the intricate interplay between risk-hedging and short-term debt. Banks optimally choose to finance their real assets using short-term debt because this incentivizes the bank to exert effort. Furthermore, because of costly equity, banks leverage to the point of just about satisfying their incentive constraints. Risk hedging through OTC contracts is thus crucial for the banks to be able to commit to high effort even with high leverage. However, this also means that the loss of a hedge, caused by the failure of the counterparty, leads investors to doubt the bank's incentives and to not roll over the bank's debt, that is, run. Contagion is not simply driven by mechanical "domino" losses in assets. The failure comes through the liability side: Investors run on banks whose hedges fail because they do not trust them. Note that this crisis is not due to the lack of liquidity in the banking system: It is due to the lack of confidence, which in turn results from the lack of bank equity. Thus, providing liquidity to the failing bank's neighbors cannot stop contagion; an increase in equity is needed, for example, through prearranged counterparty default insurance contracts described earlier.

The regulator can restore efficiency by mandating counterparty insurance. It can do so without any informational advantage over banks: It only requires the ability to tax OTC exposures and use the revenues to set up a bailout fund for the counterparties of failing banks. Alternatively, the regulator can mandate the use of a central counterparty (CCP) for hedging risk. The important feature that helps overcome inefficiency in a CCP arrangement is the establishment of a "guarantee fund" or loss mutualization: It forces all banks to contribute ex ante to bailing out counterparties of the failing bank and thereby eliminates the inefficiency because of externalities. Thus, the analysis highlights an additional benefit of central clearing beyond other benefits, such as transparency (Acharya and Bisin 2009), netting exposures (Duffie and Zhu 2011), and managing collateral. The mandatory contribution to the guarantee fund makes risk sharing through the CCP more expensive than by using OTC contracts. The result gives

5 The idea that the cost of equity matters is realistic: Froot (2001) empirically shows that insurers' equity costs do effect the premium for low-probability events.

one potential explanation of why participants in the financial market have for a long time been struggling to set up a CCP for OTC contracts: Banks have a strong incentive to opt out from a CCP and sign “cheaper” bilateral OTC contracts instead. Again, this conclusion is in line with the estimate of Singh (2010) that banks would have to raise equity capital by about $200 billion to move their trades to a CCP. A caveat is that introducing a CCP might increase moral hazard: Banks do not have an incentive to monitor their counterparties (Pirrong 2009). In an extension of the model, I show that a central counterparty might nevertheless be preferred to OTC contracting from a social point of view.

The closest paper to mine is Allen and Gale (2000), who show that interbank deposits help banks share liquidity risk but expose them to asset losses if their counterparty defaults. Their paper is different along three main dimensions. First, in their setting, the probability of counterparty default is arbitrarily small, so the crisis is socially optimal, and banks do not take potential counterparty risk into account when making decisions. Relative to Allen and Gale (2000), who solve for a social optimum, my framework allows for solving for a decentralized equilibrium with nonzero crisis probability and allows for state-contingent contracts that allow hedging against counterparty risk. In my paper, the interesting results arise exactly when the crisis is not socially optimal. Second, they follow the modeling approach of Diamond and Dybvig (1983), which does not allow for modeling the leverage choice of banks: The only choice variable of a bank is the portion of liquid assets. In my model, high leverage is an important ingredient of the crisis. Third, they model financial linkages as interbank deposits used to manage liquidity shock. My approach allows for directly analyzing the contractual choices and risk management of OTC derivatives. Also, empirical research has found that such interbank loans are unlikely to be large enough to lead to sizable contagion in developed markets (Furfine 2003; Upper and Worm 2004). There are other related papers building on the model of Allen and Gale (2000): Dasgupta (2004) allows for positive probability of crisis in a model of two banks, but he still solves for a social planner problem. Babus (2009) allows banks to hold mutual deposits to insure against the failure of a counterparty and finds a mixed strategy equilibrium in which some banks completely insure against failure, whereas others do not. However, she neither allows banks to adjust their investment choice and equity nor to buy default insurance in anticipation of a future counterparty default. Outside the context of OTC derivatives, there are other papers that show the inefficiency of the decentralized equilibrium in financial networks due to externalities: See, for example, Kiyotaki and Moore (1997), Caballero and Simsek (2009), and Zawadowski (2010).

The paper is structured as follows. The model is presented in Section 1, and the equilibrium is presented in Section 2. Section 3 explores some extensions, and Section 4 discusses some of the assumptions. In Section 5, I develop the policy implications, and Section 6 concludes.

1. Model Setup

1.1 Participants and markets

The model has three time periods: $t=0,1,2$, which I call initial, interim, and final. There are n markets on a circle indexed by $i=1...n$ and $n>3$. There are three types of market participants: n bankers, a continuum of insurance funds, and a continuum of investors. All have unlimited resources at $t=0$. The utility of bankers and insurance funds over their consumption stream is $c_0^b+\beta\cdot(c_1^b+c_2^b)$, whereas that of investors is $c_0^i+c_1^i+c_2^i$. I assume $\beta<1$. Thus, bankers want to borrow from investors because they are more impatient.[6] Only bankers can invest in risky real assets; investors can only invest in bank debt.[7] Banker i can set up a bank in market i, and no bank can operate in multiple markets. By establishing a bank, the banker becomes both manager and equityholder of the corresponding bank. All participants in the market have full understanding of the model of the economy and act rationally.

1.2 Real assets

In market i, bank i can invest a unit in a long-term real asset, which yields a return of $R_i+\epsilon_i-\epsilon_{i+1}$ at t=2, where R_i is R_H if the project succeeds and $R_L(<R_H)$ if the project fails. Whether the project succeeds depends on effort and the state of nature (discussed later). ϵ_i are bimodally distributed independent random variables realized at $t=2$: They take the value $+u$ with probability $\frac{1}{2}$ and $-u$ with probability $\frac{1}{2}$. Note that ϵ risks in neighboring markets exactly offset each other (see Figure 1). If the investment is liquidated early at $t=1$, it only yields $L(<R_H)$: I rule out partial liquidation due to the specificity of the real asset.[8] Projects also have an additional nonpledgable payoff X that is contingent on a bank settling all its contractual obligations at $t=2$. It can be thought of as franchise value or intangible capital and cannot be seized by the creditor in case of default. This conditional payoff makes the banks risk averse and induces hedging (Froot, Scharfstein, and Stein 1993); it also allows the bank to borrow, while still exerting effort (Bolton and Scharfstein 1990).

1.3 Debt

The unit investment of bank i at $t=0$ is financed by debt $D_i\geq 0$ provided by investors and equity of $1-D_i\geq 0$ provided by bankers. The maturity and interest rate on the debt contract is determined endogenously: The one period

6 This can be thought of as a modeling tool to capture the scarcity of bankers' capital.

7 Investors can be interpreted as uninformed life-cycle savers. I take it as given that they prefer simple debt securities. For a model of how debt is informationally insensitive and preferred by uninformed investors, see Gorton and Pennacchi (1990).

8 In general it would be enough to give an upper bound on liquidation value. If banks operate with high leverage, even a small loss on liquidation would erode equity faster than reducing the balance sheet and thus lead to increased leverage.

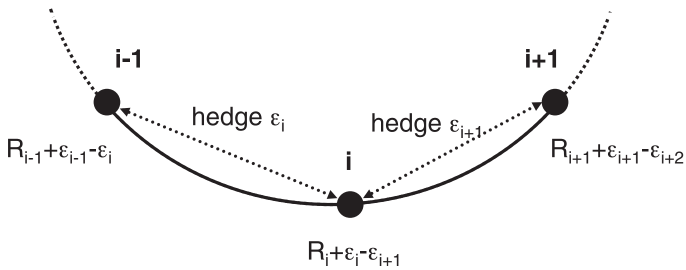

Figure 1
Market and risk structure around market *i*

borrowing rate for bank i at $t=0$ is $R_{i,l,0}$, whereas that at $t=1$ is $R_{i,l,1}$. The banker announces how much it wants to borrow and the lenders set interest rate to break even. Short-term debt has to be rolled over at $t=1$, and thus debtholders have an option to withdraw funding and force the bank to liquidate the real project. The lending decision and interest rates can be made contingent on observables (amount of borrowing, survival of certain banks) but not on the positions in OTC derivates and default insurance (see later). Most importantly, the ex post decision at $t=1$ of whether to roll over debt can depend on the indebtedness of the bank.[9] I do not allow for the renegotiation of debt contracts, given that large financial institutions usually issue bonds and commercial paper. In case there is a shortfall, all investors share equally: Thus, liquidation only happens when it is optimal for all investors. Note that there is always an inefficient bank-run equilibrium (Diamond and Dybvig 1983). Following Allen and Gale (2000), I rule out the bank-run equilibrium in case there also exists a Pareto-dominant equilibrium.

1.4 Moral hazard

Banker i makes an unobservable effort choice $e_{i,0}\in\{0,1\}$ at $t=0$ and $e_{i,1}\in\{0,1\}$ at $t=1$. The bank needs to exert effort ($e=1$) in both of the periods for the project to deliver $R_i=R_H$. Following the model of Holmström and Tirole (1997), banks receive the private benefit $B_i=B_0*(1-e_{i,0})+B_1*(1-e_{i,1})$, depending on their efforts.[10]

1.5 State of nature and outcome of the project

There are two unobservable states of nature realized at $t=1$: With probability p the unobservable state of nature is "bad"; otherwise, it is "good." The "good" state means that all projects, for which the bank exerts effort, succeed. However, in the "bad" state, the project of one of the banks chosen at random fails (delivers

[9] In a more realistic model, one can think of time $t=1$ as representing continuous rollover decisions.

[10] If a bank is liquidated at $t=1$, I assume $e_{i,1}=1$. Thus, there is only private benefit based on the effort at $t=0$.

R_L in expectation), even if the bank exerts effort.[11] Note that the shock that makes a bank's project fail regardless of its effort is mutually exclusive: Only one bank is affected. All market participants observe the evolution of all R_i's over time. From an ex ante perspective the probability of the project failing is $\frac{p}{n}$ if the bank exerts effort in both periods ($e_{i,0} = e_{i,1} = 1$).

1.6 OTC contracts

Banks can offload the ϵ risks of their real investments using OTC contracts. Only banks i and $i-1$ can contract on the risk ϵ_i, but the final realization of ϵ_i is observable and thus enforceable by the court at $t = 2$.[12] Whether banks sign contracts to share ϵ_i risks is nonobservable (and thus nonverifiable) for all market participants, except for banks i and $i-1$. To establish an ϵ hedging (OTC) contract between two banks, both banks have to agree in the initial period to enter into the contract. The equilibrium in contracting is defined using the concept of pairwise stability (Jackson and Wolinsky 1996): If a contract is established in equilibrium, neither of the contracting banks wants to terminate the contract; if a contract is not established in equilibrium, the two banks do not want to establish it. Note that the setup is symmetric, so assuming equal bargaining power, the banks do not pay each other for the hedging contract, it has price zero at $t = 0$.

1.7 Default insurance

I also allow banks to buy default insurance on other banks. The default insurance is provided by the continuum of competitive insurance funds.[13] Banks have to pay the insurance premium ex ante and the default insurances have to be fully funded: Insurance sellers have to be able to pay out all claimants in the bad state at $t = 1$. The insurance funds post a vector of prices (spreads) s, where s_j is the price at which default insurance on bank j can be purchased. One unit of default insurance contract on bank j pays out one unit if bank j defaults and zero otherwise, that is, like a digital credit default swap. Banks simultaneously choose the quantity of insurance based on the posted prices. I assume that the amount of counterparty insurance purchased by each bank is unobservable and thus the price of ϵ hedging contracts cannot be made conditional on the counterparty having default insurance on its own counterparties. In practice it is indeed hard to verify counterparty insurance, because it can easily be undone by offsetting contracts.

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[11] This large shock to one bank in the "bad" state can be thought of as a risk outside the standard model used by market participants or simply a mistake: something that cannot be hedged by the affected bank itself, but still market participants know there is some probability of it happening.

[12] One interpretation is that ex ante each bank can manipulate the probability of the two potential outcomes of ϵ_i, and only banks i and $i+1$ can detect this. This leads to a situation in which no other bank is willing to trade risk ϵ_i: a market breakdown due to asymmetric information (Akerlof 1970). This interpretation still allows the risk-sharing contract written on ϵ_i to be enforceable, because the outcome at $t = 2$ is observable to everyone, only the ex ante probability distribution is not.

[13] Letting banks sell the default insurance would just complicate the analysis without changing the main results.

1.8 Bankruptcy

Investors (debtholders) have seniority to bankers (equityholders) and receive an equal share of the proceeds from liquidating the bank. Furthermore, if the bank defaults in the interim period, all its state-contingent hedging liabilities are canceled without payment. If a bank survives until the final period once ϵ risks are realized, then it first has to settle its ϵ hedging contracts before paying back its debt. These assumptions are a reasonable approximation of the practical features of U.S. bankruptcy laws regarding derivatives (Bliss and Kaufman 2006).[14]

1.9 Definition of equilibrium

The equilibrium is defined as a subgame perfect Nash equilibrium of financing decisions and effort choices for all banks, pairwise stable OTC contracting choices, default insurance choices, interest rates, and default insurance prices s for all banks, such that bankers maximize expected payoff, the insurance fund, and investors break even.

1.10 Parameter restrictions and assumptions

The following parameter restrictions are needed to make the problem relevant to modeling counterparty risk in a contagious network:

$$R_L < L < \left(1-\frac{p}{n}\right)\cdot(R_H+X)-B_1, \tag{1}$$

$$R_H - R_L > \frac{2\cdot B_1}{1-\frac{p}{n}}, \tag{2}$$

$$B_1 \geq R_H - 1 + X, \tag{3}$$

$$B_1 - X < u < \frac{B_1}{2}, \tag{4}$$

$$\beta > \frac{1}{2} \text{ and } \beta > \frac{1-(1-p)\cdot(R_H+X-B_1)-p\cdot L}{(1-p)\cdot B_1}, \tag{5}$$

$$2u \leq B_0 < (1-p)\cdot B_1. \tag{6}$$

Restriction 1 on L ensures that it is rational for debtholders to liquidate a project when the bank does not exert effort and that the bank goes bankrupt if its debt is not rolled over in the interim period. Restriction 2 implies that it is socially optimal to exert effort. Restriction 3 ensures that banks have to keep positive equity to overcome moral hazard. Restriction 4 ensures that the ϵ risk of the bank is large enough to lead to contagion and on the other hand small enough

[14] In practice the counterparties of OTC derivatives can sue the bankruptcy estate for losses on OTC contracts and thereby become general creditors. Modeling this explicitly would complicate the analysis and is unlikely to lead to additional insights (see the discussion in Section 4).

that the bank does not want to engage in risk shifting. Restriction 5 ensures that the banker is patient enough for his participation constraint to be satisfied to invest in a bank, whereas $\beta > \frac{1}{2}$ is a technical restriction to facilitate the proofs. The upper bound on B_0 in restriction 6 implies banks' incentive constraint is binding at $t=1$, whereas the lower bound is again a technical restriction to facilitate the proofs.

2. Systemic Run and Crisis in Equilibrium

I start with stating the equilibrium in the main proposition of the paper. The rest of the section contains lemmas used for the proof of the proposition. Because banks are ex ante symmetric, the i subscript to indicate a specific bank is dropped in the rest of the paper. Proofs omitted in the text are relegated to the Appendix.

2.1 Characterization of the equilibrium

Proposition 1. If the probability of the bad state is $p \in [0, p^*)$, then there is a unique "contagious" equilibrium in which

1. banks borrow $D^* < 1$ short-term at $t=0$ at an interest rate $R^*_{l,0} > 1$ and $R^*_{l,1} = 1$
2. banks endogenously enter into OTC contracts
3. if a bank loses its OTC counterparty, it needs a debt reduction of $I = u - B_1 + X > 0$ to survive; in the absence of excess equity this can only be achieved by buying ex ante counterparty insurance
4. banks do not insure against counterparty risk using default insurance
5. the failure of a single bank in the bad state leads to a run on all banks in the system
6. if $p \in (p^{soc}, p^{ind})$, the socially optimal outcome cannot be sustained in equilibrium. In the social optimum, banks collectively set aside *2I* equity at $t=0$ and create a counterparty insurance mechanism to preempt the run, s.t. in the bad state each of the two counterparties of the failing bank receives an increase of equity of I at $t=1$.

The cutoff probabilities are related as $p^{soc} < p^* \leq p^{ind}$, more specifically,

$$p^{soc} = (1-\beta) \cdot \frac{2}{n-1} \cdot \frac{I}{R_H + X - L - (1-\beta) \cdot B_1}, \tag{7}$$

$$p^{ind} = (1-\beta) \cdot \frac{I}{\beta \cdot B_1} > \frac{n-1}{2} \cdot p^{soc}, \tag{8}$$

and the expression for p^* is given in the proof.

Proposition 1 states the existence of a "contagious" equilibrium. The sketch of the proof is the following. First, assume banks sign OTC contracts with each other, hold minimal equity (Lemma 1), and all banks collapse if one bank fails exogenously; one can think of this as a reduced form model. If the probability of the bad state is neither too high nor too low, that is, $p \in (p^{soc}, p^{ind})$, the equilibrium is inefficient. Thus, not only can banks not self-insure against their own default (Lemma 2), they are unwilling to buy socially beneficial counterparty default insurance because of the externalities in the system (Lemma 3). Second, one can show that banks are indeed willing to sign OTC contracts and thereby establish the contagion channel (Lemma 4). Note the stark contrast between how banks handle asset risk and counterparty risk (Section 2.4). Third, I prove that OTC contracts indeed act as a contagion channel. Because banks hold just enough to make the incentive constraint satisfied in the good state at $t=1$, there is no slack equity to use in the bad state. Thus, when a bank's counterparty defaults, in the absence of a equity injection, its investors deny rolling over debt to avoid it shirking. The investors of the second neighbors also become weary of the incentives of their own banks, and along a similar logic they also run, etc. Thus, in equilibrium, once the initial bank failure is known, investors of all banks linked together rationally run: a systemic freeze in lending to banks (Lemmas 5, 6, and 7). Fourth, I verify that for a range of probabilities of the bad state, $p \in [0, p^*)$, banks do not ex ante change their capital structure to avoid contagion (Lemmas 8, 9, and 10).

2.2 Leverage

Banks need to limit borrowing (i.e., hold equity) such that they have proper incentives to exert effort at $t=1$; otherwise, they face a withdrawal of their short-term debt, that is, a run. The following lemma expresses the debt limit $D^{max}(R_{l,0})$, which is decreasing in the temptation to shirk B_1 similar to Holmström and Tirole (1997).

Lemma 1. Assume a bank expects its project to deliver R_H for sure (because it is fully hedged) and it is financed by short-term debt with interest rate $R_{l,0}$. The maximum amount of $t=0$ borrowing it can roll over at $t=1$ is given by

$$D^{max}(R_{l,0}) = \frac{R_H + X - B_1}{R_{l,0}} < 1. \tag{9}$$

The expected payoff of the above bank borrowing $D^{max}(R_{l,0})$ is exactly B_1.

2.3 Private underinsurance against counterparty risk

First, I show in Lemma 2 that directly insuring the initial asset shock in the bad state is infeasible because of moral hazard. If a bank's assets are insured against adverse outcomes, it has no incentive to exert effort. Note that this also means that a policy relying on ex post bailouts creates the same moral hazard problem, and thus is not a reasonable policy in this model.

Lemma 2. It is not feasible to have an insurance scheme that aims at saving the bank hit directly (i.e., not through contagion) by low project return in the bad state.

Second, I move on to implementing a stable financial system by having banks buy default insurance on their counterparties: This way banks can hedge the risk of the initial default in the bad state of the world.[15] Lemma 3 proves one of the main results of the paper: The efficient outcome in which banks buy default insurance on their counterparties is, in general, not sustainable in equilibrium because of the externalities created by OTC contracts.

Lemma 3. If the probability of the bad state is $p \in (p^{soc}, p^{ind})$, then banks do not insure against counterparty risk, even though doing so would be socially optimal. Formally, in Proposition 1, statements (1,2,3,5) imply (4,6).

Proof. Assume the counterfactual: Banks buy default insurance of I on their counterparties at a unit cost s^{safe}. I then show that it is a profitable deviation for a bank to stop buying this insurance.

Consider the private incentive of a single bank to deviate by not buying default insurance on its counterparties.[16] The bank stays insured if and only if the payoff from staying insured and paying the cost of insurance (left-hand side) is larger than the payoff from not paying for insurance but failing more often (right-hand side) (see the Appendix for a detailed explanation of the equation):

$$\begin{aligned} &\beta \cdot \left(1 - \frac{3p}{n}\right) \cdot B_1 + \beta \cdot \frac{2p}{n} \cdot (B_1 + I) - s^{safe} \cdot 2I - \left(1 - D^{safe}\right) \\ &\geq \beta \cdot \left(1 - \frac{3p}{n}\right) \cdot B_1 - \left(1 - D^{safe}\right). \end{aligned} \tag{10}$$

From the social planner's perspective, the same trade-off (per bank) can be written as[17]

$$\begin{aligned} &\beta \cdot \left(1 - \frac{3p}{n}\right) \cdot B_1 + \beta \cdot \frac{2p}{n} \cdot (B_1 + I) - s^{safe} \cdot 2I - \left(1 - D^{safe}\right) \\ &\geq \beta \cdot (1-p) \cdot B_1 - (1 - D^*). \end{aligned} \tag{11}$$

The socially optimal outcome is not privately sustainable because banks do not internalize that buying counterparty insurance also benefits their counterparties,

[15] An additional advantage of using default insurance, instead of some insurance on assets, is that default events are relatively easy to describe and less likely to be disputed than derivatives linked to the value of assets that might be hard to value at $t = 1$.

[16] Even though the deviation is ex ante unobservable, if a bank's neighbor collapses and it does not have insurance, the investors observe that the bank does not decrease its debt, because it did not receive payments from counterparty insurance, and thus run on the uninsured bank (but not on the system as a whole).

[17] The expected payoff of investors is the same whether or not all banks insure in equilibrium, because they break even in expectation.

the counterparties of their counterparties, etc. This wedge between private and public incentives can be seen by comparing the right-hand side of Equations 10 and 11. The only difference (beyond D^{safe} vs. D^*) is the probability multiplying B_1, which is the payoff in case of survival. It is $1-\frac{3p}{n}$ in case of the private constraint and $1-p$ in case of the social constraint. From a private point of view, buying counterparty insurance only pays off if one of the two neighbors defaults, that is, with probability $\frac{2p}{n}$. From a social perspective the counterparty insurance pays off with probability $\frac{(n-1)p}{n}$: The social planner knows that all banks opt out from the insurance scheme in equilibrium, and thus every single bank fails with probability p. This difference drives a wedge between private and social incentives: a multiplier of about $\frac{n-1}{2}$ between p^{ind} and p^{soc}. The reason this multiplier is not exact is a secondary effect through the amount of debt banks can take on, that is, the difference between D^{safe} and D^*. See Appendix for the remainder of the proof and how the above equations yield the expression for p^{ind} and p^{soc}.

Thus, if $p > p^{ind}$, the probability of the bad state is high enough that even the private benefit of insuring exceeds the cost. If $p < p^{soc}$, the probability of the bad state is low enough that not even a social planner would want to set aside costly equity to avert a systemic crisis in which all banks fail. However, if $p \in (p^{soc}, p^{ind})$, the socially optimal outcome with counterparty insurance is not sustainable in equilibrium. The social efficiency of the counterparty insurance equilibrium follows from the following observations: First, insurance that prevents the initial bank failure cannot be implemented (Lemma 2).[18] Second, if $p > p^{soc}$, setting aside equity is preferred to contagion from a social point of view, and thus it is optimal to stop the contagion right away by injecting equity into the neighbors of the failing bank. Third, it is also optimal to take full advantage of the diversification benefit: Only one bank fails in the system, and thus $2I$ equity is enough to stop contagion but less is insufficient.

I now calculate the unit cost of insurance if all banks buy I amount of insurance on both of their counterparties. For the insurance to be credible, the insurance company needs to hold equity of $2I$: With probability p the bad state realizes and the insurance company has to pay I each to the two counterparties of the failing bank. If all banks fully insure, the insurance fund breaks even if and only if $\beta \cdot (2I - p \cdot 2I) - (2I - n \cdot 2I \cdot s) = 0$.[19] This yields the price of insurance in an insured (safe) system:

$$s^{safe} = \frac{1-\beta}{n} + \beta \cdot \frac{p}{n}. \tag{12}$$

[18] Even if it was implementable, one can show that $2I < R_H - R_L$. Thus, the amount of equity to be set aside to back up counterparty insurance is less and thereby would be socially preferred to saving the failing bank directly.

[19] The first term is the expected amount that remains in the insurance fund at $t=2$ and can be consumed by the insurer, whereas the second term is the amount the insurance fund has to set aside at $t=0$ (beyond the premium paid) to have $2I$ equity.

The first term of the insurance is due to the cost of prefunding the insurance (cost of equity), whereas the second term is the actuarially fair price equal to the present value of the expected payout. ■

Counterparty insurance is a public good and the inefficiency is due to free riding.[20] The amount of equity $2I$ that needs to be set aside to fund a counterparty insurance scheme (i.e., the cost) does not depend on the number of banks that contribute to the costs by buying the insurance. Thus, even if a bank does not contribute, it can still enjoy most of the benefits of a stable system; it only loses the insurance against its immediate neighbors, which go bankrupt with probability $\frac{2 \cdot p}{n}$. The multiplier 2 comes from the fact that each bank has two counterparties: Thus, the incentives depend on how many counterparties each bank has (see Section 4).

The missing market that makes the privately optimal choice socially inefficient is that OTC contracts cannot be made contingent on the counterparty's decision about counterparty default insurance. This is realistic, because the derivative trading book of banks is unobservable, and even if it were, it would be extremely difficult for an outsider to sign a contract contingent on whether a bank has hedged the default of its counterparty. This is exactly the sense in which OTC contracts in the model are "nonexclusive": Banks cannot control all other contract choices of their counterparties, and thus banks might sign other contracts on the side undermining the efficient allocation (Bisin and Guaitoli 2004). However, in a network setting, neither would simply allowing for banks to contract on their counterparties' insurance choice lead to an efficient outcome nor would "due diligence" contracts (see Section 3.3). Another manifestation of the same missing market is the assumption that the sellers of default insurance cannot price their insurance contingent on the counterparty default insurance choice of all other banks. This assumption is made implicitly because of the competitive way default insurance is priced: Sellers post prices, and buyers buy at this given price.

Both the social planner and the banks are only willing to insure counterparty risk if the probability of the bad state p is high enough. The reason is that equity is costly: The per unit cost is $1-\beta$ because insurance sellers (like equityholders) are impatient and want a high return on equity. The cost of equity is what shows up in the first term of the price of default insurance in Equation 12 and then becomes an important term in the cutoff probabilities for buying insurance in Equations 7 and 8. Remember that I assume that the CDS has to be prefunded, that is, collateralized, by insurers (or bank equityholders) using the risk-free security. I thus rule out contractual arrangements in which investors (bondholders) provide state-contingent assets to insurers that would allow for a cheap way of backing (collateralizing) the default insurance. This is a

[20] Thus, my paper also suggests an alternative explanation for the breakdown of private coinsurance arrangements between banks described in Calomiris (2000): If banks are connected in a network, coinsurance collapses because of externalities.

reasonable assumption for two reasons: First, in practice almost all derivatives have to be collateralized by cash or government bonds.[21] Second, average investors typically only buy highly rated bonds but do not engage in derivatives, such as selling CDS contracts.

2.4 Risk sharing through OTC contracts

Because investors cannot observe the OTC contracts of the bank, banks could potentially gain from risk-shifting on their ϵ risks. In the following, I establish that if the size of the ϵ risks of the projects are limited from above (restriction 4), banks have the proper incentives to hedge these risks. The main driver of this behavior is that banks are not risk neutral: The continuation value X is received at $t=2$ but is only conditional on repaying all debt. The following lemma highlights the strong incentive to be entangled in OTC hedging contracts: Investors can rely on the banks' incentives to hedge their ϵ asset risks. For a detailed discussion on the observability assumption, see Section 4.

Lemma 4. In equilibrium, any bank i completely hedges asset risk ϵ_i with bank $i+1$ and risk ϵ_{i-1} with bank $i-1$.

Proof. The bank hedges both of its ϵ risks if and only if[22]

$$\beta\cdot(1-p)\cdot B_1 > \beta\cdot(1-p)\cdot\left[\frac{1}{4}\cdot 0+\frac{1}{2}\cdot B_1+\frac{1}{4}\cdot(B_1+2\cdot u)\right]. \tag{13}$$

The left-hand side is the payoff from hedging in the contagious equilibrium. The three terms on the right-hand side are the following: With probability $\frac{1}{4}$ the real project yields R_H-2u, the bank suffers a run and goes bankrupt, yielding zero to the banker; and with probability $\frac{1}{2}$ the two ϵ risks cancel out, and the bank receives the same payoff B_1 as if it had hedged; with probability $\frac{1}{4}$ the real project yields R_H+2u, and thus the bank gains $2u$ additional payoff from risk shifting. The above equation is equivalent to $B_1>2\cdot u$, which is satisfied by restriction 4. See Appendix for the remainder of the proof. ∎

In the model, the reason banks do not necessarily want to shift risk is the presence of the continuation value X (similar to the hedging motive in Froot, Scharfstein, and Stein 1993). In the proof of Lemma 4, $u<\frac{B_1}{2}$ from restriction 4 ensures banks do not want to shift asset risk. Combining this with the other half of restriction 4, that u has to be big enough to lead to bankruptcy $u>B_1-X$, this result implies $X>\frac{B_1}{2}$. That is, X has to be at least half of the final

[21] See ISDA Margin Survey 2012. Available at: www2.isda.org/functional-areas/research/surveys/margin-surveys.

[22] The bank does not change the amount it borrows to not reveal the deviation. Thus, it faces the same interest rate in both cases. Also, note that although X, the continuation value, induces hedging it does not directly show up in the Equation 13 because it is hidden in $B_1=R_H-R_{l,0}D+X$, which contains the payoff of the bank in period $t=2$ along with the continuation value X.

payoff of the bank to make sure the bank does not want to shift risk. Thus, this model helps understand why banks hedge some risks, while leaving other risks unhedged: Hedging counterparty risk entails sacrificing costly equity to avoid low-probability counterparty risk.

Lemmas 3 and 4 make it clear that banks treat some risks differently from others. On the one hand, banks are willing to enter into OTC contracts with each other to reduce asset risk. On the other hand, they are not willing to hedge counterparty risk, even though doing so would be socially optimal. Thus, the main insight of these two lemmas is that OTC contracts themselves endogenously transform asset risk into low-probability counterparty risk, which creates systemic risk exactly because of the presence of the OTC contracts.

In the model, the two main factors driving underinsurance against counterparty risk are the cost of equity and the low-probability of realization of the bad state. Buying default insurance is potentially too costly compared to its benefits. The cost of setting aside equity is independent of the probability of the realization of the bad state. However, the benefit of insuring only materializes in this low-probability state on the order of p. Thus, the incentive to insure counterparty risk is low in the first place, and the externalities in the entangled system further reduce the incentives leading to inefficiency: Banks fail to insure against counterparty risk. This is why only a relatively high $p > p^{ind}$ makes the private benefits of insurance outweigh the private costs.

Counterparty risk is different from asset risk for several reasons. First, within the model, the probability of adverse realization of asset risk is much higher. Whereas the OTC risk sharing contracts on asset risk are very likely to benefit the bank, insurance on counterparty risk rarely does: only in the low-probability bad state and even then only if one of the bank's counterparty fails. Note that upon counterparty default, a bank faces the same ϵ risk as it faced ex ante. Thus, counterparty risk is not more "severe" in terms of potential left tail outcomes than asset risk. Second, outside the model, covariance has a role: In practice, counterparty risk usually materializes in states of the world in which equity is already low. In these states, losing a counterparty can have a more severe effect on a bank and it is also expensive to set aside equity for this state. See Section 4 on how macroeconomic risk could be incorporated into the model. Third, counterparty risk has a double role that is not true for other low-probability risks. It is not only a risk, it is also the means of contagion: If a bank does not insure its counterparty risk, the adverse effect of this is transmitted to others through the counterparty risk it poses to other banks. If a bank does not manage another type of low-probability risk, this might make it fail, but it will not spread to other banks without them having an unhedged counterparty exposure to the failing bank.

Underinsurance against counterparty risk is not traditional risk shifting toward lenders. In a simple model of risk shifting toward lenders, banks would want to shift risk regardless of the type of risk: asset risk, counterparty risk, etc.

The reason is that in the margin it is always worth shifting a bit more risk, and the ex ante probability of the bad realization has no effect. If banks want to shift risk, they would never want to engage in any risk-sharing contracts ex ante, and thus there would be no "entangled financial system" and no systemic risk. Also, the right-hand side of Equation 10 reveals that the bank cannot capture the upside of ϵ risk and shift the downside risk to bondholders. The reason is that if the bad state realizes it is revealed that it did not insure counterparty risk and its lenders run at $t=1$. However, the benefits of risk shifting would only materialize at $t=2$ in the bad state.

Nevertheless, one can think of underinsurance against counterparty risk as a broader type of risk shifting toward counterparties. By not insuring, the bank gains on the upside by saving on the high cost of insurance (because of setting aside costly equity). However, on the downside if the bad state realizes and its neighbor fails, the bank fails and imposes losses on its counterparties. Note that the counterparties are not creditors (ϵ is only realized at $t=2$) but still lose because the loss of their counterparty renders their equity insufficient.

2.5 Contagion

I now show that if all banks borrow up to $D^{max}(R_{l,0})$ at $t=0$, then all banks collapse in the bad state. First, I show that the bank, whose project fails in the bad state, goes bankrupt (Lemma 5). Second, I show that if bank i collapses and leaves its counterparty, bank $i+1$, unhedged, that counterparty also suffers a run of investors, and thus it fails (Lemma 6). Because all banks are linked, investors of all banks rationally run.

Lemma 5. If a bank borrows $D^{max}(R_{l,0})$ at $t=0$, it goes bankrupt if its debt financing is not rolled over at $t=1$. If bank i has a low expected realization R_L of its real project, then its debt financing is not rolled over either.

The intuition is that it is optimal for creditors to demand a failing project to be abandoned and liquidated because the payoff is still better than letting this bad project mature and receive $L > R_L$. This is true regardless of whether the project is failing because of bad luck or shirking. In fact, this is exactly the rationale for short-term debt: Creditors want to have the right to terminate a project if it is surely going to have a very low return, for example, when the bank did not exert sufficient effort in the initial period.

The next step, formalized in Lemma 6, is at the heart of the contagion mechanism. If all banks borrow up to $D^{max}(R_{l,0})$, the failure of a bank's OTC counterparty leads to potential losses and a violation of its incentive constraint. Because it would choose to shirk, its creditors run and it is preemptively liquidated.

Lemma 6. If a bank borrows up to $D^{max}(R_{l,0})$, it exerts full effort at $t=1$ if both of its counterparties survive. However, it chooses to shirk if one or both of its counterparties defaults, leaving it unhedged. This results in its debt not rolling over at $t=1$.

When a counterparty fails, the incentives of the bank can be restored by injecting equity, that is, reducing its debt. In Lemma 7, I calculate the size of the necessary equity injection I.

Lemma 7. The amount of debt reduction needed at time $t=1$ to stabilize a bank that lost one of its counterparties is $I=u+X-B_1>0$.

Clearly the higher the potential loss u from the loss in counterparty, the more equity that has to be injected in the counterparty. However, I is slightly less than u because at $t=2$ the bank still has an equity buffer of B_1-X (assuming $\epsilon=0$) that it can lose and still be able to repay its debt. Lemma 7 also highlights that restriction 4 is crucial in making the system contagious: u has to be large enough $u>B_1-X$, s.t. banks need additional equity to survive the collapse of a counterparty. It also follows that the default insurance sellers have to set aside funds of $2I$ to be able to stop contagion in the bad state: pay out I to both counterparties of the failing bank. Note that in this case equity is negative debt, and thus the bank can use the equity injection I to retire some of its debt and roll over less. This lower debt ratio is observed by investors, and thus they are willing to roll over the debt of the bank even though it lost a hedge due to its neighbor's default.

2.6 Existence and uniqueness of the equilibrium

In the following lemmas I verify that there are no profitable deviations from the contagious equilibrium and the equilibrium is unique if $p\in[0,p^*]$. Because these lemmas are more technical in nature, the formal exposition of the lemmas and their proofs are relegated to the Appendix. In Lemma 8, I prove that long-term debt is not a profitable deviation. Short-term debt allows for more borrowing because it makes it easier for the bank to commit to high effort: Investors can withdraw in a state-contingent manner if the bank's project is failing.[23] In Lemma 9, I show that deviating to holding higher levels of equity is also not profitable. In Lemma 10, I show uniqueness by ruling out equilibria with heterogenous equity holdings.[24] The formal proof of Proposition 1 can be found in the Appendix.

[23] Because I focus on the inefficiencies due to the network of hedging contracts, I choose to model short-term debt as an efficient choice (Calomiris and Kahn 1991; Diamond and Rajan 2001) instead of an inefficient one (Brunnermeier and Oehmke 2012).

[24] Note that the existence of an equilibrium with two or more banks holding more equity would still leave the system contagious and inefficient, because for the efficient outcome to be sustainable in equilibrium one needs $p>p^{ind}$. The reason is that for banks to have an incentive to set aside costly equity for the bad state, they need to be directly affected by the bad state with a relatively high probability. This relatively high probability effect comes from part of the system collapsing in the bad state.

2.7 Numerical example

For illustrational purposes I give a numerical example for which all the parameter restrictions and assumptions are satisfied and the unique equilibrium is the contagious one. I calibrate the model to annual quantities. I use $R_H = 1.01$ to model that the assets of banks are not much more profitable than their liabilities. I set $n = 15$ to model the core of the global financial system. The nonpledgeable payoff is set to $X = 0.10$, reflecting that the stock market value of large banks is small compared with their balance sheets. The return to early liquidation is set to $L = 0.9$, the outcome with the failing project to $R_L = 0.7$. The private benefit in the short run is $B_0 = 0.1$ and higher in the long run $B_1 = 0.13$. The size of the ϵ risks is set to $u = 0.05$. The bankers' discount factor is $\beta = 0.85$, implying expected returns of about 18%.[25] The probability of the bad state is chosen to be $p = 1\%$, corresponding to the bad state (potential crisis) once a century. This means the financial system is contagious because the probability p above which it is socially optimal to have counterparty insurance is $p^{soc} = 0.22\%$, whereas banks choose to buy counterparty insurance only once the probability of the bad state rises above $p^{ind} = 2.7\%$. One can verify that $p^* = 1.83\%$, and thus the contagious equilibrium is unique if $p \in (0, 1.83\%)$. If $p \in (1.83\%, 2.37\%)$, the contagious equilibrium exists but is not unique, and there is another inefficient equilibrium with at least two banks self-insuring by issuing less debt. This illustrates the large wedge between social and private incentives. The social planner would want banks to insure against bad states that occur about every 500 years, whereas the banks would only choose to insure if the bad state happens about every forty years.

3. Extensions of the Baseline Model

3.1 Collateral

Another way of protecting against counterparty failure beyond default insurance or autarky is to avoid any exposure by ex ante demanding collateralization of OTC positions. I assume that the choice of collateralization is unobservable for lenders like all other risk management choices of the bank. The version of the model presented in the paper does not have credit exposure: The value of the ϵ contracts are not due to credit exposure here but are due to the value of avoiding bankruptcy at $t = 2$. Still, one can interpret collateral in the framework of the model: Banks have to set aside funds (e.g., in a third-party account) that are transferred to the counterparties in case of a default. The following proposition shows that collateralization is prohibitively expensive because banks substantially need to limit borrowing ex ante at $t = 0$.

[25] This value might seem high, but it is a long-term return over more than a year, and the average annual return on common equity for investment banks was 16% before the crisis. See "Capital spenders" in "A special report on international banking," *The Economist*, May 19, 2007.

Proposition 2. No pair of banks chooses to deviate from a contagious system to collateralizing their OTC ϵ hedging contracts. Furthermore, if $p < p^{ind}$, a system with collateralized OTC contracts is not sustainable in equilibrium. It is also socially inefficient to collateralize positions: buying counterparty default insurance is always superior from a social perspective.

Collateralization is socially wasteful because lots of extra equity is stored in the system ex ante, which is not put to use, and only one of the banks fails in the bad state. Counterparty default insurance leaves much less extra equity in the system but then deploys it in a state-contingent manner to the banks whose counterparty failed. Counterparty default insurance takes advantage of the diversification benefit: Not all banks fail at the same time. In the model, only one bank fails exogenously, in reality there are diversification benefits in as much as initial bank failures are not perfectly correlated. The result of Proposition 2 is in line with the fact that OTC contracts between large creditworthy banks and dealers are in general not subject to collateral (Singh 2010).

3.2 Monitoring of counterparties

One argument against any kind of counterparty insurance (or CCP) is that it undermines banks' incentives to monitor their counterparties (Pirrong 2009). In this subsection, I extend the model to allow for costly counterparty monitoring to evaluate this argument. Assume that for a cost c a bank can monitor its left counterparty and thereby decrease the probability of its exogenous failure (real project realization R_L if it exerts effort) from $\frac{p}{n}$ to $\frac{p'}{n}$, where $p' < p$. The insurance sellers do not have access to the monitoring technology because they are not counterparties of the bank.

Proposition 3. In a contagious system, banks monitor if the cost is sufficiently low: $c < \frac{p-p'}{n} \cdot \beta B_1$. The contagious system (which induces monitoring) is socially preferred to the system in which banks buy counterparty insurance if and only if

$$p' < p^{soc} + \frac{p - p^{soc}}{n} - \frac{c}{R_H + X - L - (1-\beta) \cdot B_1}. \tag{14}$$

Thus, allowing for a system to be contagious just to induce banks to monitor their counterparties is only socially optimal if the monitoring technology is very efficient. Monitoring has to be able to decrease the probability of counterparty default close to the probability at which having a crisis is socially optimal. Thus, despite the incentive effect on monitoring, counterparty insurance (or a CCP) is socially optimal for a wide range of the probability p of the bad state. Note that banks only monitor if the cost is low enough that it is privately optimal to do so. This means that they might not monitor with higher costs c even though this might be socially optimal.

3.3 Bilaterally demanding counterparty insurance

I extend the model to show that banks cannot overcome inefficiency by demanding counterparty insurance from each other. I allow banks to demand that the counterparties buy counterparty insurance on their other counterparties when signing an ϵ hedging contract. For example, the contract between banks $i-1$ and i prescribes that bank i has to buy counterparty insurance on bank $i+1$ (Figure 1). Because a deviation from pairwise stable equilibrium allows for bilaterally establishing a new contract, I allow banks to bilaterally renegotiate this provision if that is strictly beneficial for both.

Proposition 4. Demanding counterparty insurance from one's counterparty is not an equilibrium if $p < p^{ind}$.

The intuition is that if a bank already has insurance on its counterparty, it does not care anymore whether that counterparty is managing its own counterparty risk (assuming the insurance funds cannot monitor counterparty insurance choices). Proposition 4 shows that the missing market is more complicated: It is not enough to simply allow banks to demand their counterparties to buy default insurance on their own counterparties. One needs to allow insurance funds to price default insurance conditional on the counterparty insurance choice of *all* banks. In this case, if a bank allowed its counterparty to become uninsured, the insurance funds would then double the price of the default insurance that bank $i-1$ bought on bank i because the deviation doubles the default probability of bank i from $\frac{p}{n}$ to $\frac{2p}{n}$. This would exactly offset all gains of bank i from not buying insurance on bank $i+1$ because it is priced based on a default probability $\frac{p}{n}$ of bank $i+1$. Given that there are no benefits (only costs), the two banks would not deviate, and thus the insured system would be an equilibrium regardless of the value of p. Such a private contract that prices default insurance conditional on all banks' choices is unlikely to be enforceable in reality.

4. Discussion

4.1 OTC contracts in bankruptcy

For simplicity, I assumed no credit exposure between banks because this maintains symmetry between banks. In the model, counterparty default leads to losses because of the impossibility to replace contracts. Although credit exposure might add to the losses from counterparty failure, I argue that replacement costs are also an important channel. In reality, the replacement value of OTC contracts may far exceed the mark-to-market credit exposure of the contract, especially in cases in which large amounts of contracts are replaced in the time of market stress. The reason is that the mark-to-market value is priced based on the ex ante price of buying an additional marginal unit of the OTC contract, whereas the ex post market price after the failure might be substantially different if a large amount of contracts are bought. Evidence from the aftermath of the Lehman default suggests that even if counterparties could rehedge, they incurred substantial losses,[26] because they had to rehedge in a very volatile market with counterparties who were less willing to hedge their risks. Also, many risks are specialized and thus illiquid, making them hard to replace (Stulz 2010; Froot 2001).

I also assumed that losses due to counterparty default cannot be recouped if they are not collateralized. In reality there is likely to be at least some recovery. Although collateral is not subject to automatic stay in bankruptcy, losses beyond the collateral have to be litigated (Bliss and Kaufman 2006). This means that OTC counterparties become general creditors of the failing firm. Although assuming partial recovery of losses does weaken the contagion mechanism, other factors such as an involvement in a litigation process with uncertain outcome can enforce contagion (the contagion mechanism of the model relies exactly on uncertainty about future exposure). Also, if one assumes that the run on the counterparty of the failing bank is not socially efficient, the recovery value L on the bank might be very low. In the case of Lehman Brothers, the recovery on derivatives claims after years of litigation was below forty cents on the dollar.[27]

4.2 Macroeconomic risk

The model has only purely idiosyncratic risk. One may introduce macrorisk by adding a common shock to the final payoff of the real project. Because banks' incentives are affected by the final payoff of the real project, the bankers have to limit borrowing and thereby keep a higher equity stake to make sure their incentives are aligned with those of the investors even in bad macrostates. On the one hand, the effect of such a higher equity buffer is that in better realizations of the macrostate, banks might have surplus equity that allows them to survive the collapse of their counterparty. On the other hand, in bad realizations of the macrostate, the additional ex ante equity buffer is depleted and the contagion mechanism outlined in the paper is at work again. Thus, it is in bad macrorealizations that the outlined mechanism is most likely to work and it is exactly in such states when the replacement of OTC contracts is likely to be the most costly due to the aggregate lack of equity (Shleifer and Vishny 1997).

4.3 More general network structure

The circle of OTC contracts created by the banks is a specific network. However, the main intuition regarding inefficiency carries over to networks in which each bank has only a few counterparty exposures that are large enough such that the failure of these counterparties leads to a run on the bank itself. On the other

26 "Citadel files claim to recover \$470.5 million," *The Wall Street Journal*, August 25, 2009.

27 "Lehman creditors in pay-out settlement," *Financial Times*, June 29, 2011.

hand, a network in which each bank has the same large exposures to every other bank, does not allow for free-riding: If one bank fails, all others are directly exposed and have to hold their own counterparty insurance to be able to survive. If one bank fails and another one did not buy insurance, it will also fail because it cannot simply free-ride on the insurance of others. There is very limited amount of data about cross-bank exposures in the OTC market. Under congressional pressure AIG reported its counterparties, which for CDS's is very concentrated: The top three counterparties accounted for half of the exposure, whereas other major banks had very little exposure to AIG.[28]

4.4 Observability

The observability of OTC hedging and default insurance choices of the banks is not essential for the main results. This holds in as much as the contracting choices are not contractable and inforceable, because the latter would complete the market. The observability of counterparty insurance would make interest rates contingent on these choices, but this would not fully eliminate the externality, and in equilibrium banks would still choose not to insure against counterparty failure. Note that because investors could observe the ϵ hedges, they would not need to trust banks to hedge, thereby making Lemma 4 and the upper bound on u in restriction 4 superfluous.

5. Policy Implications

5.1 Taxing OTC contracts

In principle, one can attain the social optimum by mandating counterparty default insurance on OTC contracts, but this might be hard to implement. However, the government can implement the same transfer scheme as that implemented by the counterparty default insurance without having to verify who holds default insurance on whom (i.e., without any informational advantage). All it has to do is collect taxes on OTC contracts (the price of counterparty insurance), set up a bailout fund, and in case of a default inject equity into the counterparties of the failing bank to avoid the run of the investors. Not only is such a scheme relatively simple to implement but it also compensates the government for its commitment of capital.

5.2 Central clearing of OTC contracts

Another possible policy is mandating the use of a central counterparty (CCP), which entails loss mutualization. This would ensure that all banks need to set aside equity in the so-called guarantee fund of the CCP to cover losses from the default of a bank in the system. If a bank fails, the CCP has two options: hold the

[28] Source: AIG, March 15, 2009, from Attachments A, B, and D. Available online (last accessed September 21, 2009): www.aig.com/aigweb/internet/en/files/CounterpartyAttachments031809_tcm385-155645.pdf.

OTC contracts until maturity or close the contracts out by giving appropriate payments to the counterparties of the failing bank. In the former case, the CCP would need to have at least $2u$ equity in the guarantee fund to be able to pay out u to both counterparties in case the realization of the ϵ shocks are such. The latter case is exactly equivalent to the socially efficient counterparty default insurance scheme: The closeout sum given to the two banks would be I each. It is more cost efficient than holding the contracts until $t=2$ because it requires equity of only $2I<2u$. Thus, the main results from default insurance carry over to a CCP. One concern is the way CCP's operate: If they demand full collateralization of all positions, this is similar to the setup of the model with collateral, which I show is socially inefficient in Proposition 2.

Thus, in my paper, the role of the CCP is to eliminate ex ante externalities: It forces banks to contribute to counterparty insurance and also provides the mechanism to do so in the form of a guarantee fund. This is a further benefit of the CCP that is different from the following advantages proposed in other papers (for a summary, see Stulz 2010). First, the CCP evenly spreads the impact of a failure ex post.[29] Second, the CCP allows for monitoring in opaque markets: Acharya and Bisin (2009) show that participants sell more insurance on certain states of the world than what they can honor and the CCP can limit exposures. Third, the CCP allows for netting: Duffie and Zhu (2011) show a CCP would help reduce costly margin accounts by netting. Fourth, a CCP reduces the gross exposures by allowing offsetting contracts between three or more counterparties to be canceled (for more on gross vs. net exposures, see Oehmke and Zawadowski 2012).

5.3 Banning OTC contracts

Because I also model the ex ante formation of OTC contracts, one can also evaluate the effect of banning risk-sharing contracts altogether. Banning ϵ hedging contracts is socially suboptimal if there is some mechanism to implement counterparty insurance (see above). The reason is that banning OTC contracts forces banks into autarky and to keep higher levels of equity. Thus, this policy cannot take advantage of the diversification effect that not all banks fail at the same time and equity set aside for the bad state can be deployed in a state-contingent matter. One can show that for large number of banks n, banning OTC contracts is socially suboptimal if $p\in(0,p^*)$: Welfare in the contagious system is higher.[30] One must add that if a systemic crisis has additional nonmodeled negative externalities in the economy, banning OTC contracts might be a second-best policy if counterparty default insurance or a bailout fund is not implementable.

[29] This is similar to the result of Allen and Gale (2000), who show that a more complete network is more resilient, because the impact of the initial failure has a smaller effect on each counterparty.

[30] The proof is similar to that of Lemma 9; the interval in which the statement is true might slightly change for low n.

5.4 OTC contracts in bankruptcy

The assumption of this paper is that OTC counterparties are only shielded from losses if they collateralize their exposures or buy default insurance on their counterparties. In reality, noncollateralized losses can be litigated and at least partially recovered. Thus, if OTC counterparties were completely shielded, that is, they would promptly (and without any uncertainty) receive the full replacement value of their OTC contracts, crisis would not spread. This is exactly the reason that the regulator has given de facto seniority to derivatives by exempting collateral from automatic stay. However, because the seniority only applies to collateralized positions, contagion is still possible, especially because I have argued that collateralization is not chosen by banks and wasteful from a social perspective. Furthermore, some legal scholars and economists have questioned the desirability of the preferential treatment of collateral (Edwards and Morrison 2005; Roe 2011; Bolton and Oehmke 2011).

5.5 Limiting counterparty exposures

Regulators have also tried to preempt contagion by limiting banks' exposure: Section 165(e) of the Dodd-Frank Act limits banks' credit exposures to any unaffiliated company at 25% of its equity. If banks can diversify their counterparty exposure at a relatively low cost, limiting exposures might indeed be an optimal policy. In the context of the model, this means decreasing u such that the bank has enough equity to pay off its obligations for sure at $t=2$ to preempt a run (s.t. restriction 4 is not satisfied). However, if the banks cannot easily choose another counterparty, like in the model, counterparty limits force the banks into autarky, which is inefficient (see Lemma 9 and Proposition 1). Also, focusing only on credit exposure might be too narrow in cases in which replacement costs are high (see Section 4).

6. Concluding Remarks

This paper uses a model to show that bilateral contracting in OTC markets does not in general lead to an efficient outcome. The reason is that OTC contracts create both a contagion channel and also an externality. The contagion makes idiosyncratic risk systemic, while the externality inhibits banks from privately implementing the socially optimal outcome using counterparty insurance to eliminate systemic risk. Thus, the model helps explain the lack of counterparty risk management (such as collateralization) among large banks and also gives an explanation of why it is difficult to establish and sustain the central clearing of derivatives. This paper analyzes the effect and limitations of different policies to overcome inefficiency within the model.

Appendix: Proofs and Additional Lemmas

Proof of Lemma 1.
The incentive constraint of a bank surviving at $t=1$ and holding no risk can be written as following:

$$\beta\cdot\left[R_H - R_{l,0}\cdot D + X\right] \geq \beta\cdot B_1. \tag{A1}$$

The left-hand side is the payoff if the bank chooses to exert effort $e_1=1$, and the right-hand side is if it chooses $e_1=0$ (the initial equity choice is taken as given and its effect on payoff omitted). The pledgeable payoff is $R_H - R_{l,0}D$, the nonpledgeable is X for sure, because the bank does not hold any risk. Note that because there is no risk involved in lending at $t=1$ implying $R_{l,1}=1$. The above equation delivers the expression for $D^{max}(R_{l,0})$ and restriction 3 implies $D^{max}(R_{l,0})<1$. The incentive constraint implies that the expected payoff (before discounting) of the banker that borrows $D^{max}(R_{l,0})$ is B_1 at $t=2$, conditional on survival at $t=1$. ■

Proof of Lemma 2.
Assume there is an insurance scheme in place that guarantees bank i's survival until $t=2$, even if its real project is observed at $t=1$ to deliver only R_L. Banks know this ex ante, and thus if they choose to shirk in both periods they receive a payoff of B_0+B_1. Note that the insurance allows them to survive at $t=1$ and shirk again. On the other hand, if they decide to exert effort in both periods, they receive an expected payoff of B_1 and that only conditional on the good state (Lemma 1). Thus, the payoff from shirking already at $t=0$ is higher than not because $B_0\geq 0$. This means there cannot be an equilibrium in which banks have insurance on the realization of R_i at $t=1$ being R_L. ■

Proof of Lemma 3. (continued from main text)
Payoffs from deviations. In Equation 10, the first term on the left-hand side is the payoff to the insured bank if neither it nor one of its neighbors defaults. The bank's payoff conditional on survival is B_1 (from Lemma 1). The second term is the payoff if one of its neighbors defaults: In this case it receives payoff I from the insurance. The third term is the forgone consumption due to the price of the counterparty insurance; the fourth is the forgone consumption of the banker at $t=0$ to contribute equity ($E=1-D$) to the real project. As for the right-hand side of the equation, if the bank is uninsured but all the others are insured, it goes bankrupt if either it or one of its two counterparties goes bankrupt in the bad state, thus with probability $\frac{3p}{n}$. For now just assume that if financing is withdrawn at $t=1$, then the equityholder does not receive anything (see Lemma 5).

Amount of borrowing. In a system in which all banks buy counterparty insurance, the system is stable and not contagious. Denote the amount borrowed by D^{safe} and the interest rate $R^{safe}_{l,0}$ on the short-term loan at $t=0$ is set such that investors break even:

$$\left(1-\frac{p}{n}\right)\cdot R^{safe}_{l,0}\cdot D^{safe}+\frac{p}{n}\cdot L = D^{safe}. \tag{A2}$$

Combining this with the maximum amount that can be borrowed (Lemma 1), yields

$$D^{safe} = \left(1-\frac{p}{n}\right)\cdot(R_H+X-B_1)+\frac{p}{n}\cdot L. \tag{A3}$$

If all the banks decide to not buy insurance and hold the minimum amount of equity to roll over debt, the system is contagious, that is, in the equilibrium described in Proposition 1. Investors anticipate the equilibrium, and the breakeven interest rate is determined by

$$(1-p)\cdot R^*_{l,0}\cdot D^* + p\cdot L = D^*. \tag{A4}$$

Combining this with Lemma 1 yields

$$R^*_{l,0} = \frac{1}{1-p\cdot\left(1-\frac{L}{R_H+X-B_1}\right)}, \tag{A5}$$

$$D^* = \frac{R_H+X-B_1}{R^*_{l,0}} = (1-p)\cdot(R_H+X-B_1)+p\cdot L. \tag{A6}$$

Equations 7 and 8 for p^{soc} and p^{ind} follow from Equations 10, 11, 12, and the above expressions for D^{safe}, D^*. The relationship $p^{ind} > \frac{n-1}{2} \cdot p^{soc}$ follows from restriction 1.

Asymmetric equilibria. Up to now we have only compared symmetric equilibria in which either everyone insures or no one insures against counterparty risk. I show that asymmetric equilibria in which at least two banks insure while others do not, can be ruled out if $p \in [0, p^{ind})$. Assume that not all banks insure against counterparty risk but at least two do. More specifically, although bank i insures, the next insured bank "to the left" is $i+m$; thus, there are $m-1$ uninsured banks between these two. If bank i wants to buy insurance on $i+1$, the insurance is priced to take into account that, conditional on bank i not failing, bank $i+1$ fails with probability $\frac{m}{n} \cdot p$. This is because it fails if any of the m banks $\{i+1, ..i+m\}$ fail. Similarly, the next insured bank "to the right" is $i-m'$. Assuming a pricing of default insurance that linearly depends on the default probability, this increases the price of default insurance on bank $i+1$:

$$s = m \cdot \frac{1-\beta}{n} + m \cdot \beta \cdot \frac{p}{n}. \tag{A7}$$

Bank i will not deviate from such an equilibrium by not buying insurance on bank $i+1$ if and only if

$$\begin{aligned} &\beta \cdot \left(1 - \frac{p}{n}\right) \cdot B_1 + \beta \cdot \frac{(m+m') \cdot p}{n} \cdot I - \left(\frac{1-\beta}{n} + \beta \cdot \frac{p}{n}\right) \cdot (m+m') \cdot I - (1-D^i) \\ &\geq \beta \cdot \left(1 - \frac{(m+1) \cdot p}{n}\right) \cdot B_1 + \beta \cdot \frac{m' \cdot p}{n} \cdot I - \left(\frac{1-\beta}{n} + \beta \cdot \frac{p}{n}\right) \cdot m' \cdot I - (1-D^i). \end{aligned} \tag{A8}$$

This results in exactly the same value of p^i as before. Thus asymmetric equilibria exist if and only if the full insurance symmetric equilibrium is sustainable. For the analysis when only one bank insures (i.e., self insures), see Lemma 9. ■

Proof of Lemma 4. (continued from main text)
Hedging only one of the ϵ risks also yields a lower payoff:

$$\beta \cdot (1-p) \cdot B_1 > \beta \cdot (1-p) \cdot \left[\frac{1}{2} \cdot 0 + \frac{1}{2} \cdot (B_1 + u)\right], \tag{A9}$$

which holds if and only if $B_1 > u$, which again is satisfied by restriction 4. Note that the payoff of only partially hedging ϵ risks cannot be higher than not hedging at all because it limits risk shifting: Thus, we can conclude banks do not shift risk by not hedging.

We assumed up to now that the bank goes bankrupt at $t=2$ if the realization of its unhedged risk is $\epsilon = -u$. This is true if the bank cannot repay its debt at $t=2$:

$$R_H - u < R^*_{l,0} \cdot D^*(R^*_{l,0}). \tag{A10}$$

Substituting for $R^*_{l,0} \cdot D^*(R^*_{l,0})$, one arrives at $B_1 - X < u$, which is exactly restriction 4. Trivially, if a bank fails when it loses $-u$ on its unhedged ϵ exposure, so will it when it loses a total of $-2u$ on both unhedged exposures. ■

Proof of Lemma 5.
A bank goes bankrupt at $t=1$ if the liquidation value of the project is not enough to cover debt obligations, that is, $R_{l,0}D > L$. If the minimum level of equity is held, implying a debt level of $R_{l,0}D^0(R_{l,0}) = R_H + X - B$, the condition for bankruptcy is $R_H - B + X > L$, which in turn is satisfied by restriction 1. For the bank, the project of which fails in the bad state, regardless of effort, the expected return on the investment is R_L regardless of the bank's effort at $t=1$. Bankruptcy follows from restriction 1, that is, that $R_L < L$, and the investors optimally liquidate the real investment. ■

Proof of Lemma 6.
If neither of the counterparties default, exerting effort is optimal by Lemma 1. If one of the counterparties defaults, the bank is either owed u by the defaulting neighbor with probability $\frac{1}{2}$ or owes the defaulting bank u with probability $\frac{1}{2}$. Whether or not the bank lost money on the derivative is known to the bank itself but not to the investor. If the bank lost money on the ϵ hedging contract, the banker's incentive constraint is violated even with $R_{l,1}=1$, thus it shirks, and the project delivers R_L. Because by Lemma 5 the banker is wiped out in bankruptcy, all of the payment R_L accrues to the investors. Denote the amount of debt to be rolled over at $t=1$ by D_1. Knowing this, the investor demands higher interest rate $R_{l,1}$ to break even:

$$\frac{1}{2}\cdot R_{l,1}\cdot D_1+\frac{1}{2}\cdot R_L=D_1 \tag{A11}$$

resulting in

$$R_{l,1}=2-\frac{R_L}{D_1}>1, \tag{A12}$$

where the inequality follows from Lemma 5 that the banker cannot pay back its debt in bankruptcy, that is, $D_1>R_L$. This higher interest rate affects banks that do not lose money by their counterparty's default, changing their incentive constraint. Now effort can only be sustained if the amount of debt D_1 to be rolled over at $t=1$ satisfies

$$\beta\cdot\left[R_H-R_{l,1}\cdot D_1+X\right]\geq\beta\cdot B. \tag{A13}$$

Substituting $R_{l,1}$ we arrive at

$$D_1\leq\frac{1}{2}\cdot(R_H+X-B)+\frac{1}{2}\cdot R_L. \tag{A14}$$

If the bank originally borrowed $D^0(R_{l,0})$, the amount it has to roll over at $t=1$ is $R_{l,0}\cdot D^0(R_{l,0})=R_H+X-B$.

If two of the counterparties default, the banker loses $2u$ with probability $\frac{1}{4}$; otherwise, it does not lose on the default of a counterparty. The bank's incentive constraint is clearly violated if it borrowed up to $D^0(R_{l,0})$ and lost $2u$ on the two OTC contracts; thus, it shirks and the project fails. The interest rate $R_{l,1}$ thus has to increase for investors to break even, resulting in the incentive constraint to be violated even if the bank does not lose on the OTC derivatives. ■

Proof of Lemma 7.
If a bank loses one ϵ hedge at $t=1$, it can only roll over its debt if its incentive constraint is not violated. This can be insured in two dierent ways: (1) by injecting enough equity into the bank to make sure it can pay back its debt even if it loses u at $t=2$ because of the missing hedge and (2) by injecting enough equity to offset the loss in continuation value of $\frac{X}{2}$. Note that the bank defaults with probability $\frac{1}{2}$ because it does not default if the real project delivers R_H+u. In the first case the binding constraint is a solvency constraint at $t=2$:

$$R_H-u-R_{l,0}\cdot D^{max}(R_{l,0})+I_1\geq 0. \tag{A15}$$

which yields $I_1=u+X-B_1$. We used that the bank survives for sure at $t=2$, implying $R_{l,1}=1$. In the second case the binding constraint is the incentive constraint at $t=1$:

$$R_H+\frac{X}{2}-R_{l,0}\cdot D^{max}(R_{l,0})+I_2\geq B_1, \tag{A16}$$

which yields $I_2=\frac{X}{2}$. We implicitly used the breakeven constraint of the lenders implying that the bank needs to pay them $R_{l,0}\cdot D^{max}(R_{l,0})$ in expectation. Restrictions 3 and 4 imply $I_2>I_1$; thus, even the lower amount I_1 is enough for the bank to survive for sure $t=2$, and no bank needs an equity injection of I_2 to roll over its debt, implying $I=I_1$. $I>0$ follows from restriction 4. ■

Lemma 8. If $p<p^{term}$, bankers do not deviate from the contagious equilibrium of Proposition 1 by issuing long-term debt. Furthermore, $p^{term}>(n-1)\cdot p^{soc}$.

Proof of Lemma 8.

If a bank borrows long term, the banker can borrow up to the point to make him indifferent between long-term borrowing and shirking in both periods to collect B_0+B_1. Note that with long-term lending, investors cannot liquidate the bank under any circumstances. Because shirking in only the second period is clearly suboptimal, the banker shirks in both periods, and the expected payoff as of $t=0$ is B_0+B_1, and thus the payoff in the good state must be at least B_0+B_1:

$$\left(1-\frac{p}{n}\right)\cdot\left(R_H-R^{term}\cdot D^{term}+X\right)+\frac{p}{n}\cdot B_1 \geq B_0+B_1. \tag{A17}$$

Lenders break even in expectation at $t=0$:

$$D^{term}=\left(1-\frac{p}{n}\right)\cdot R_l^{term}\cdot D^{term}+\frac{p}{n}\cdot R_L, \tag{A18}$$

where we used several facts. First, that the bank shirks if its real project is expected to deliver R_L, which follows from restrictions 2 and 6. Second, that the bank can repay all its debt even if it is unhedged and loses on both ϵ risks, that is, $R_H-2u-R^{term}\cdot D^{term}\geq 0$, which follows from restrictions 3 and 6. We also used the fact that the bank exerts effort even if the whole system collapses, i.e. $R_H-R^{term}\cdot D^{term}+X\geq B_1$, which follows from restrictions 3 and 6. These equations yield R_l^{term} and D^{term}.

Now let us compare the payoff in the contagious system and the payoff if the bank deviates to long-term borrowing. The banker chooses not to deviate in a contagious system if

$$\beta\cdot(1-p)\cdot B_1-(1-D^c)\geq\beta\cdot(B_0+B_1)-(1-D^{term}), \tag{A19}$$

yielding

$$p^{term}=(1-\beta)\cdot\frac{n}{n-1}\cdot\frac{B_0}{R_H+X-L-(1-\beta)\cdot B_1+\frac{\beta B_1+R_L-L}{n-1}}>(n-1)\cdot p^{soc}, \tag{A20}$$

where the relation to p^{soc} results from restrictions 2, 4, and 6. ■

Lemma 9. If $p\in(0,p^{aut})$, no bank has an incentive to deviate from the contagious equilibrium of Proposition 1 by holding more equity (borrowing less). Furthermore, $p^{aut}>\frac{n}{4}\cdot p^{soc}$.

Proof of Lemma 9.

First, note that ex ante, a bank always weakly prefers hedging, even if it has enough equity to roll over its debt if it loses its hedges. The reason is that if a bank hedged and lost that hedge, it is in exactly the same situation as if it had not hedged its ϵ risk at all ex ante. In Lemma 4 we have already shown that the bank does not want to shift risk. Furthermore, a given level of debt hedging allows the bank to survive with weakly higher probability and collect the continuation payoff X.

There are three types of autarky. In the first, which I call risky autarky, the bank borrows only to the level that it can still roll over its debt at $t=1$ if its neighbors collapse but does not necessarily survive at $t=2$. In the second, which I call safe autarky, the bank survives at $t=2$ even if both of its ϵ risks are realized such that the payoff of the real project is R_H-2u. In the third type, which I call full autarky, the bank survives even if its real project only delivers R_L. I analyze all three types of autarky in order.

1. Risky autarky. If the bank chooses risky autarky, it needs enough equity to survive even if both its counterparties collapse. Note that autarky entails borrowing less, so it is an observable deviation from the contagious equilibrium and thus affects the interest rate. Autarky is risky in the sense that the bank cannot pay back its debt in full if the real

project delivers only $R_H - 2u$, which happens with probability $\frac{1}{4}$. The relevant incentive constraint is

$$R_H + \frac{3}{4}X - R_{l,0}^{r.aut} \cdot D^{r.aut} \geq B_1. \tag{A21}$$

Note that as of $t=1$, $R_{l,1}$ adjusts such that the bank needs to pay $R_{l,0}^{r.aut} \cdot D^{r.aut}$ in expectation to its creditors. The breakeven condition for investors at $t=0$ is

$$D^{r.aut} = \frac{p}{n} \cdot L + \left(1 - \frac{p}{n}\right) \cdot R_{l,0}^{r.aut} \cdot D^{r.aut}. \tag{A22}$$

We also have to show that borrowing $D^{r.aut}$ allows the bank to pay back its debt at $t=0$ if the final payoff of the project is R_H, that is, the two ϵ risks are exactly offsetting s.t. that $R_H - R_{l,1}^{r.aut} \cdot R_{l,0}^{r.aut} \cdot D^{r.aut} \geq 0$. This follows from restrictions 3 and 4 combined with the breakeven condition for investors at $t=1$, which can be written as (assuming the banker can indeed repay if the two risks are offsetting):

$$R_{l,0}^{r.aut} \cdot D^{r.aut} = \frac{1}{4} \cdot (R_H - 2u) + \frac{3}{4} \cdot R_{l,1}^{r.aut} \cdot R_{l,0}^{r.aut} \cdot D^{r.aut}. \tag{A23}$$

A banker decides not to deviate to risky autarky in a contagious system if and only if

$$\beta(1-p) \cdot B_1 - (1-D^*) \geq \beta(1-p)\left[R_H + X - R_{l,0}^{r.aut} \cdot D^{r.aut}\right] + \beta \frac{n-1}{n} p \cdot B_1 - (1 - D^{r.aut}). \tag{A24}$$

The left-hand side is the payoff from the contagious system. The first term on the right-hand side is the payoff in the good state: The banker has to repay $R_{l,0}^{aut} \cdot D^{aut}$ in expectation and will survive at $t=2$ for sure. The second term is the payoff in the case of the bad state, which is exactly B_1 because the incentive constraint is binding. This delivers $p^{r.aut}$:

$$p^{r.aut} = (1-\beta) \cdot \frac{n}{n-1} \cdot \frac{\frac{X}{4}}{R_H + X - L - (1-\beta) \cdot B_1 - \frac{n\beta - 1}{4(n-1)} X} > \frac{n}{4} \cdot p^{soc}. \tag{A25}$$

The relation to p^{soc} follows from restriction 4 and $\beta > \frac{1}{4}$ from restriction 5.

2. Safe autarky. In the case of safe autarky, the bank needs enough equity such that it can pay back even if its real project yields $R_H - 2u$:

$$R_H - 2u - R_{l,0}^{s.aut} \cdot D^{s.aut} \geq 0. \tag{A26}$$

In this case the bank survives unless it is directly affected by the low return R_L, because combining restrictions 2 and 4 implies $R_H - 2 \cdot u > R_L$.

Thus, the investors' breakeven condition is

$$\left(1 - \frac{p}{n}\right) \cdot R_{l,0}^{s.aut} \cdot D^{s.aut} + \frac{p}{n} \cdot L = D^{s.aut}. \tag{A27}$$

Combining these we arrive at the maximum amount of debt an autarkic bank can borrow at $t=0$:

$$D^{s.aut} = \left(1 - \frac{p}{n}\right) \cdot (R_H - 2u) + \frac{p}{n} \cdot L. \tag{A28}$$

Conditional on surviving at $t=1$ in a contagious system the payoff to the banker is B_1, given that the incentive constraint of the bank is binding. If the bank is in autarky, the expected payoff conditional on surviving at $t=1$ is $R_H + X - R_{l,0}^{s.aut} \cdot D^{s.aut} = B + 2u$, where we used the incentive constraint. In a contagious system, the bank does not deviate to autarky if

$$\beta \cdot (1-p) \cdot B_1 - (1-D^*) \geq \beta \cdot \left(1 - \frac{p}{n}\right) \cdot (R_H + X - R_{l,0}^{s.aut} \cdot D^{s.aut}) - (1 - D^{s.aut}). \tag{A29}$$

The equation for $p^{s.aut}$ can be written as the following:

$$p^{s.aut} = (1-\beta) \cdot \frac{n}{n-1} \cdot \frac{2u + X - B_1}{R_H + X - L - (1-\beta) \cdot B_1 + \frac{1-\beta}{n-1} \cdot (2u + X - B_1)} \geq \frac{n}{2} \cdot p^{soc}, \tag{A30}$$

where the inequality follows from restrictions 1, 3, and 4 and $\beta > \frac{1}{4}$ from restriction 5.

Note that for risky autarky to indeed lead to a potential default at $t=2$, $R_H - 2u < R_{l,0}^{r.aut} \cdot D^{r.aut}$ has to hold, which is equivalent to $2u > B_1 - \frac{3}{4}X$. Using $\beta > \frac{1}{4}$ one can easily show that if this condition is violated, that is, $2u < B_1 - \frac{3}{4}X$, then $p^{s.aut} < p^{r.aut}$. Thus the bank would deviate to safe autarky instead of risky autarky. Thus, we can conclude that the nonexistence of risky autarky does not influence our analysis, because it would not be the preferred deviation.

3. Full autarky. Banks could potentially also self-insure by limiting their borrowing in a way that they can still repay their debt at $t=2$ for sure, even if the project realizes only the low return R_L. Because this debt is risk-free, the banker can borrow up to R_L. However, it means keeping an even larger equity buffer than safe autarky because we have shown that $R_H - 2\cdot u > R_L$; thus, if a bank can repay its debt in case of low outcome, it can also repay it if it loses both its hedges but is owed on both of them. Comparing the payoff with that from the contagious system, the bank surely prefers to stay in the contagious equilibrium and not deviate if
$$\beta\cdot(1-p)\cdot B-(1-D^*)\geq\beta\cdot\left(1-\frac{p}{n}\right)\cdot(R_H+X-R_L)+\beta\cdot\frac{p}{n}\cdot X-(1-R_L). \qquad \text{(A31)}$$
The cutoff probability for self-insurance p_{si} can be approximated from below as follows
$$p^{f.aut}=(1-\beta)\cdot\frac{R_H+X-B_1-R_L}{R_H+X-L-(1-\beta)\cdot B_1-\frac{\beta}{n}(R_H-R_L)}>(n-1)\cdot p^{soc}, \qquad \text{(A32)}$$
where the lower bound of $R_H+X-B-R_L>2I$ follows from restrictions 2 and 4.

All three types of autarky. Combining the cutoff values for safe autarky, risky autarky, and self insurance one arrives at
$$p^{aut}=min(p^{s.aut},p^{r.aut},p^{f.aut})>\frac{n}{4}\cdot p^{soc}. \qquad \text{(A33)}$$
■

Lemma 10. An equilibrium in which at least two banks hold more equity (borrowing less) exists only if $p>p^{2.aut}$. Furthermore, $p^{2.aut}>\frac{n-1}{2}\cdot p^{soc}$.

Proof of Lemma 10.
If at least two banks hold enough equity to survive in the bad state, they only need to hold enough equity to withstand the collapse of one of their counterparties, because in general only a part of the system collapses that is bordered by two banks with sufficient equity. Note that surviving the collapse of two counterparties would take us back to the risky autarky of Lemma 9. The relevant constraint is that the bank does not go bankrupt at $t=2$, even if it loses u, due to the loss of an ϵ hedge:
$$R_H-u+X-R_{l,0}^{2.aut}\cdot D^{2.aut}\geq 0. \qquad \text{(A34)}$$
The breakeven condition for investors at $t=0$ is
$$D^{2.aut}=\frac{p}{n}\cdot L+\left(1-\frac{p}{n}\right)\cdot R_{l,0}^{2.aut}\cdot D^{2.aut}. \qquad \text{(A35)}$$
A banker is willing to borrow less than in the contagious equilibrium if and only if
$$\beta(1-p)\cdot B_1-(1-D^*)\geq\beta\left(1-\frac{p}{n}\right)\left[R_H+X-R_{l,0}^{2.aut}\cdot D^{2.aut}\right]-\beta\cdot\frac{p}{n}\cdot\frac{X}{4}-(1-D^{2.aut}). \qquad \text{(A36)}$$
The left-hand side is the payoff from the contagious system. The first term on the right-hand side is the payoff unless the bank is hit directly in the bad state; the banker has to repay $R_{l,0}^{aut}\cdot D^{aut}$ in expectation and will survive at $t=2$ for sure. The second term is a correction for the case when the other autarkic bank is hit by the shock directly. In this case the bank loses both ϵ hedges, that is, goes bankrupt at $t=2$ with probability $\frac{1}{4}$ but can roll over its debt at $t=1$. This delivers $p^{2.aut}$:
$$p^{2.aut}=(1-\beta)\cdot\frac{n}{n-1}\cdot\frac{u+X-B_1}{R_H+X-L-(1-\beta)\cdot B_1+\frac{(1-\beta)I-\beta\frac{X}{4}}{(n-1)}}>\frac{n-1}{2}\cdot p^{soc}. \qquad \text{(A37)}$$
The relation to p^{soc} follows from restrictions 1, 3, and 4 and $\beta>\frac{1}{2}$ from restriction 5. ■

Proof of Proposition 1.
Statement (2) follows from Lemma 4. The short-term nature of debt in (1) follows from Lemma 8, the amount borrowed from Lemma 1, whereas the equilibrium interest rate is expressed in Lemma 3. (5) follows from Lemmas 5 and 6 along with the following observation. If a bank's debt is not rolled over, it goes bankrupt; thus, the investors of the neighboring banks should not roll over their debt either. Because all banks are connected, by induction investors of all banks should run if they observe one bank failing (the first failing bank has a payoff of R_L on its real project). (3) follows from Lemma 7, whereas Lemma 3 proves that (1,2,3,5) imply (4) and (6).

The fact that the system proposed in (1)–(6) is a unique equilibrium is proved by showing that there are no profitable deviations and there are no other equilibria. Lemma 9 proves that decreasing the amount of borrowing at $t=0$ is not profitable if $p < p^{aut}$. Lemma 8 shows that no bank finds it profitable to deviate to borrowing long term if $p < p^{term}$. The uniqueness of the equilibrium if $p \in (0, p^*)$ is proven by Lemma 3, showing that a system with default insurance is not an equilibrium, and by Lemma 10, showing that it is not an equilibrium when either that two or more banks revert to autarky. Lemma 2 proves that saving the bank hit by the bad state is not a feasible equilbrium. The participation constraint as of $t=0$ is satisfied because the utility in the equilibrium is positive by restriction 5. Conditional on the good state, the incentive constraint as of $t=1$ is satisfied by Lemma 1, whereas that of $t=0$ is satisfied if $\beta(1-p)\cdot B_1 \geq \beta \cdot B_0$ because the payoff of the banker in the good state is B_1, and the payoff in the bad state is 0. The inequality is satisfied by restriction 6 .

Thus, the contagious system is an equilibrium and unique if and only if $p \in (p, p^*)$, where p^* is given by

$$p^* = min(p^{ind}, p^{aut}, p^{2.aut}, p^{term}) > \frac{n}{4} \cdot p^{soc}. \tag{A38}$$

■

Proof of Proposition 2.
Full collateralization would mean setting aside collateral of u for each counterparty. I only consider partial collateralization, in which a bank gives its counterparty $I < u$, because this is also enough to ensure that the counterparty can survive the bank's collapse. Note that because the choice of collateralization is unobservable, the bank cannot borrow more to cover collateral because lenders would take that as borrowing above the maximum debt amount. Thus, the bank needs to cover collateral using its equity.

First, consider the contagious system and the potential deviation in which two banks choose to bilaterally set aside collateral of I. Note that the cost of doing this is the same as the full systemic cost of counterparty default insurance, which entails setting aside equity of $2I$. However, the benefits cannot exceed that of counterparty insurance, which delivers a safe system. Thus, banks do not have the incentives to deviate if $p < p^{ind}$. Second, consider the system in which all banks set aside equity of $2I$ (I for each counterparty). This means $n \cdot 2I$ equity set aside in the system, which is clearly inefficient, because counterparty default insurance only requires equity of $2I$. Also, if banks choose to deviate from counterparty default insurance if $p < p^{ind}$, they also deviate from collateralization, because the benefits are the same, but the costs are n times higher. ■

Proof of Proposition 3.
A bank monitors its counterparty if and only if its private utility is higher with monitoring than without. Monitoring reduces the probability of counterparty default by $\frac{p-p'}{n}$, and the expected payoff in case of survival is B_1, which yields the expression in the proposition.

The social utility is higher in the contagious system with monitoring if and only if

$$\beta \cdot \left(1 - \frac{3p}{n}\right) \cdot B_1 + \beta \cdot \frac{2p}{n} \cdot (B_1 + I) - s^{safe} \cdot 2I - (1 - D^{safe}) \leq \beta \cdot \left(1 - p'\right) \cdot B_1 - (1 - D^*) - c, \tag{A39}$$

which yields the expression in the proposition. ■

Proof of Proposition 4.

Assume the counterfactual, that it is an equilibrium that all banks buy counterparty default insurance and also demand that their counterparties buy insurance. Now consider bank $i-1$, which demands that bank i buy counterparty insurance on bank $i+1$. To show that the equilibrium contracting is not pairwise stable, consider the following deviation: banks $i-1$ and i bilaterally renegotiate their contract, allowing for i to not buy counterparty insurance on $i+1$. From Lemma 3 this is a strictly profitable deviation for bank i if $p < p^{ind}$. Because the deviation is strictly profitable, bank i can offer an arbitrarily small positive amount (bribe) to bank $i-1$. Note that the deviation (i.e., not buying insurance) is not observable. Thus, the insurance funds cannot raise the price of counterparty insurance of bank $i-1$ that it bought on bank i. Hence, an arbitrarily small bribe is enough to prompt bank $i-1$ to deviate, because it is originally indifferent: It already has insurance on bank i; thus, it is shielded from bank i failing due to the failure of bank $i+1$. Thus, there is a strictly profitable deviation if $p < p^{ind}$, proving the proposition. ■

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